

# ANALYSIS OF QUEUE-SIZE BEHAVIOUR AND THROUGHPUT OF A SYSTEM WITH BUFFER CONTROLLED BY A ROPE AND PRODUCTION SPEED CONTROLLED BY A DRUM

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**Abstract:** The original contribution of the paper is the consideration a flow shop system with setup and closedown times of a critical machine. In addition, the critical machine suffers from random failures. In the paper various concepts of increasing throughput by adjusting the bottleneck buffer by a rope and controlling speed of production by a drum are presented according to the Drum-Buffer-Rope (DBR) concept. Various capacities of the machine tied to the rope are given. The case of the production process that must be suspended when the buffer reaches the limit, should be avoided. Similarly, the case of the production process which a machine is not effectively used and needs to be stopped, is not desirable. The objective of the paper is to analyze the impact of the burstiness of input data on the DBR system performance. A sufficient size of the constraint buffer to protect the bottleneck against processing time variations and system disturbances is established.

**Key words:** Drum-Buffer-Rope, Theory of Constraints, managing buffers, flow shop, bottleneck, critical machine.

## 1. INTRODUCTION

The complexity of production cells depends on a mix of products with various volumes and operations characteristics. The flow of products (usually one-piece flow) is controlled by kanban or conwip methods. One-piece flow works well when the operations within the cell are well balanced, there are relatively small set-up times between various products. Moreover, there is little randomness, processing time variation and transport times between operations take low percentage of operation cycle time (Millstein and Martinich, 2014). For such conditions, the Drum-Buffer-Rope (DBR) method is an alternative.

The term of DBR can be brought closer starting from the early 1980s, when numerous authors have attempted to apply the Theory of Constraint (TOC) for the systems. The TOC core relies on the identification and use of system constraints. In 1986,

Goldratt and Fox implemented the TOC at the operational level by proposing the concept of the Drum-Buffer-Rope (DBR) (Goldratt and Fox, 1986). In the DBR method, inventory movement is controlled under a pull production concept (Lambrecht and Segaert, 1990; Watson and Patti, 2008). The drum states as the constraint (the bottleneck) whose schedule sets the rate of production for a production system. The bottleneck is connected to the gate operation by a rope. Materials are released at a rate that depends on material consumption at the bottleneck. The rope ensures a constant inventory level between the bottleneck and the gate (material release). A buffer is placed before the bottleneck in order to protect the bottleneck output against statistical fluctuations.

There are two possibilities of controlling the DBR system: by means of a time buffer or work in process (WIP) buffer (Goldratt, 1990; Gonzales et al., 2010). *The time buffer* sets an input rate while the WIP upstream of the bottleneck fluctuates depending of stochastic characteristic. Input rate must be updated periodically to adjust to the rate dictated by the bottleneck to prevent infinite WIP. *The WIP buffer* depends on the bottleneck meanwhile input rate fluctuates. Upstream WIP is limited by the bottleneck which sends kanban or conwip cards to release the jobs in the systems.

The DBR systems are not free from complexity, especially when each product has a different bottleneck. The complexity of DBR systems increases even more when bottlenecks feed each other or the routes of several jobs run through the bottlenecks. The DBR problem includes buffer sizing, batch sizing and material release time computations. Due to a dynamic feature of real systems the possibility of bottleneck shifting increases the problem. Disruptions such as machine failure (especially the bottlenecks), fluctuations in

setup and processing times influence a system throughput.

Basing on the DBR concept various modifications of the method are proposed. Various scenarios of DBR systems are investigated due to differences in processes or randomness. A brief review of the DBR concept applications is given below.

Chang and Hung, (2014) researched the DBR problem by searching a priority rule to dispatch jobs through the bottleneck. The authors treated the drum as the due-date assignment method, the rope as the order release rule and the buffer as a dispatching rule for a bottleneck machine in a flow shop system. They proposed various priority rules based on buffer status to dispatch jobs. The priority indicators might be distorted due to the bottleneck failure. For each job, the number of capacity hours of the bottleneck adding half of a production buffer which is needed to meet the due date was established. Additionally, the result was multiplied by  $(1-\alpha)$ ,  $\alpha$  value was lower when the disruption of the bottleneck affected the front of the route than the end of the route. The jobs were indexed based on the non-increasing permutation of the capacity plus half buffer. The jobs were reindexed observing the influence of the random breakdown situations of the bottleneck over the due dates meeting condition. The objective was to keep at least 80% of a given due date performance indicator in the event of a random failure. Gonzalez et al., (2010) also investigated the influence of dispatching rules over the performance of a shop floor system controlled by the DBR method. Following dispatching rules were tested: SPT (Shortest Processing Time), SRPT (Shortest Remaining Processing Time), SI (SI Truncated), EDD (Earliest Due Date), LS (Least Slack), CR (Critical Ratio), FCFS (First Come First Served) and SRO (Select in Random Order). The performance of the priority rules for three criteria was evaluated: average tardiness, maximum tardiness and WIP. Robustness of criteria using Taguchi signal-to-noise ratio was evaluated for a range of manufacturing scenarios. EDD, SI and FCFS were the most robust priorities for varying processing and set-up times and breakdowns. Taguchi robustness assessed globally which dispatching rule best copes with all possible changing conditions.

The core of the DBR concept involves buffers insertion and sizing. Buffers protect production systems against fluctuations caused by internal disruptions. Ye and Han (2008) classified three types of buffers: capacity, time and stock buffer. *The time buffer* constituted a period of time to protect downstream production against the idleness of the bottleneck due to upstream disruption. *The constraint buffer* constituted an optimal number of items waiting before the bottleneck and protected it

against the idleness. According to Louw and Page (2004) adding the time buffer was better solution than inventory buffers because there were not tied to specific parts. Golmohammadi (2015) implemented the TOC rules for job shop systems. He built Master Production Schedule (MPS) for the system which included more than one bottleneck. The critical machine which had the largest difference between its actual and required capacity was the primary constraint. A detailed schedule was created using the DRB concept. Two buffers were placed: between the raw material release stage and the primary constraint. Initially, the author assumed the batch size to be equal to the demand. The buffer size was assumed to be the summation of processing and setup times from the first machine to the constrain. Simulations revealed the hidden effect of queues and WIP. In fact, other machines acted as bottlenecks. He investigated the impact of setup times, the impact of products those do not use the critical machine (free products) on throughput, the impact of priority rules in machine assignment to free products. He assumed that there were no defective products and machine failure. Finally, he determined the input values for the system that would generate the best machine utilization, profit and the number of finished products. Ye and Han (2008) introduced a constraint buffer, an assembly buffer and a shipping buffer to a production system. The assembly buffer was located after the bottleneck and was also fed by a non-constraint resource without waiting. The shipping buffer was located at the end of the production line in order to ensure due dates. Each upstream machine may have failed with different rate. MTTR had different percentage of influence on the buffer size. They noticed the relationship between the constraint and assembly time buffers and a number of feeder machines. They observed the throughput of the system with various number of feeder machines.

The TOC is dedicated to batch production. The smaller batch sizes the more throughput and profit. The smaller batch size pushes more material to the end of a production system and reduces WIP (Golmohammadi, 2015). Ideal batch size is one piece flow, which is typical for Kanban systems. Watson and Patti (2008) compared the impact of buffering under Just in Time (JIT) and TOC on a five-station cell performance. Additionally, unplanned machine failure was possible except for the bottleneck. The impact of two buffering techniques on the indicators was investigated: total output, lead time and standard deviation of lead time. The Kanban system placed a small buffer of WIP between each machine/work center. The material was pulled through the system in order to minimize the size of inventory. The DRP system

maintained the constant flow of material and level of WIP between the bottleneck and the gate. Comparing analysis indicated that the DRB system accommodated greater levels of variability, produced more with less inventories. Thanks to the constraint buffer, the bottleneck was robustly protected against potential failures of downstream machines. Millstein and Martinich (2014) used the advantages of one-piece flow, transfer-batch sizing and DBR method to develop Takt Time Grouping (TTG) method. A customer lot size was divided into equal TTGs - smaller transfer batches. A transfer-batch size for each product depended on the maximum operation cycle time for the bottleneck. TTG quantity was computed by dividing the Tempo by the min value of maximum operation cycle times of products. The Tempo determined how often one TTG leaves the system to complete the deadline and the order quantity (lot). Processing times were approximately the same at bottlenecks for each transfer batch. Kanbans controlled the movement of a transfer batch called “piece” to adopt the one-piece flow. The Tempo was iteratively adjusted to the group sizes in order to reduce batch cycle time variation relative to the mean and achieve flowtime and WIP objectives. The TTG method worked well when there were multiple moving bottlenecks.

Most heuristic scheduling techniques according to the TOC use ad hoc simulations in order to observe the behaviour of a system and the impact and the role of input variables on the system performance. Most popular variability sources are processing times (Ye, 2008; Watson, 2008; Gonzalez, 2010; Chang, 2014), breakdowns (Watson, 2008; Gonzalez 2010; Chang 2014) and set-up times (Gonzalez, 2010; Krenczyk and Olender, 2014), batch size (Ye, 2008; Golmohammadi, 2015), inter-arrival time between batches, release time for raw materials (Golmohammadi, 2015), different number of feeder machines (Ye, 2008). A task is stored in a finite-capacity buffer if the critical machine is busy or setup activities are done. The influence of buffer capacity should be also taken into account. Additionally, whenever a production system is empty, the last machine (bottleneck) is stopped by a closedown time. A machine needs a setup time before providing the service of the first task after the idle time. Such a production system has limited throughput depending on the bottleneck efficiency including processing, set-up and close-down times. Thus, the variability of the above parameters in terms of the DBR throughput should be investigated.

In this paper, a flow shop system with setup/closedown times exponentially distributed is considered. Productions tasks enter the system with exponentially distributed interarrival times and are served by times assumed to be exponentially

distributed. Arriving tasks form a single waiting line and are served in the order of their arrivals. The critical machine suffers from random failures. The production system is optimized using the Drum-Buffer-Rope concept. In the paper various concepts of increasing throughput by adjusting the bottleneck buffer by a rope and controlling speed of production by a drum are presented.

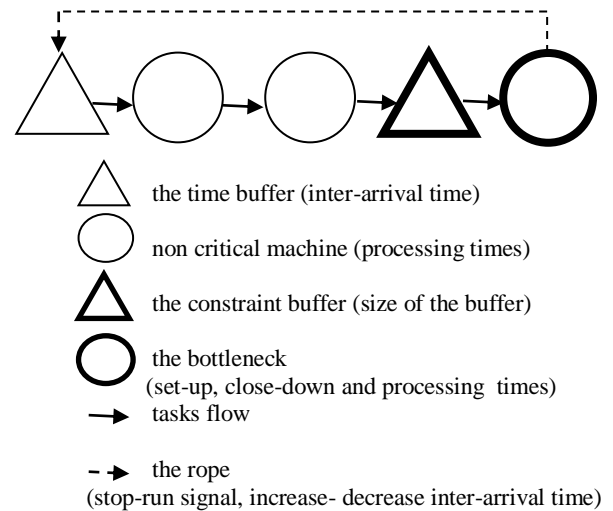


Fig. 1. M/M/1/N tandem queue model with set-up and close-down times of the bottleneck and finite-capacity constraint buffer

Various capacities of the machine tied to the rope are given. The case of the production process which must be suspended when the buffer reaches the limit, should be avoided. Similarly, the case of the production process, which a machine is not effectively used and needs to be stopped, is not desirable. Different scenarios of the drum beaten at the various speed of the bottleneck are considered and the inventory level and throughput are observed. The production system efficiency is computed, beating a drum at the speed of the last machine. This case is preferred by a demand-driven pull type production system. The comparative analysis is carried out for various failure-free, repair, setup and close-down times.

The objective of the paper is to analyze the impact of the burstiness of input data on the production system performance. The exemplary conclusion is noticed, as processing time of a single job increases the throughput of the DBR system decreases. As processing time increases, the differences of throughput are getting smaller and smaller. The explanation of this phenomenon is in that, the production system stabilizes since when the rope is triggered by the bottleneck. The increase of interarrival time has little impact on throughput of the DBR system, since, a lot of jobs will be lost due to the constrain buffer overflow and production rate dictated by the bottleneck.

## 2. PROBLEM FORMULATION

As presented in Figure 1, an M/M/1/N tandem model of the DBR system with a single bottleneck and finite-capacity constraint buffer is considered. The monitoring both the flow of input and output tasks is essential in the performance evaluation and optimal utilization of a DBR system.

Orders are accepted for realization by the bottleneck on a FIFO (first-in-first-out) basis until the DBR system becomes empty. If the bottleneck finds no task in the constraint buffer it enters the close-down phase. If a task arrives at the DBR system before the close-down period expires, the bottleneck immediately goes back to the busy state to execute the task. If no task arrives during the close-time phase, the bottleneck goes to the idle phase. If the bottleneck finds one or more tasks waiting in the constraint buffer it begins the execution of the task/s after a setup period only for the first task. The example of the queueing system with a single resource considering setup and close-down times and finite-capacity waiting rooms one can find in (Niu et al. 2003). The transitions among the close-down, set-up and processing times are presented in Figure 2.

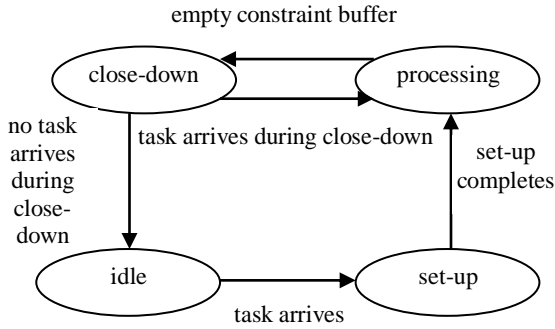


Fig. 2. Phase transition diagram of the DBR model with close-down and set-up times

One of important stochastic characteristics which can be used in such a monitoring is departure process, that at any fixed time moment  $t$  takes on a random value  $h(t)$  equal to the number of tasks completely processed till this moment. The observation and the analysis of sensitivity of the process counting successive successfully processed jobs on changing “input” parameters of the DBR system, such as intensities (parameters) of exponentially distributed interarrival time ( $\lambda$ ), processing time ( $\mu$ ) of a single task, set-up time ( $\alpha$ ), close-down time ( $\beta$ ), time of failure-free operation ( $\gamma$ ) and repair time ( $\eta$ ), may provide useful information for optimization of the DBR system. A number of jobs lost due to the constraint buffer overflow and given capacity of the bottleneck need to be minimized. The utilization level of the uper-flow machines needs to be adjusted to the level of the bottleneck in order to reduce WIP. The prediction of the time of real (actual) sojourn time (waiting times + processing times) of a task waiting in the constraint

buffer helps to make a decision about producing in cooperation.

The objective is to analyze the impact of burstiness of input data on the DBR system performance. Availability of the bottleneck and throughput ( $TR$  - the number of jobs completely processed) of the DBR system are evaluated for various input data. The in-depth numerical study on the sensitivity of the constraint buffer-size distribution and the bottleneck efficiency on changes of  $\lambda, \mu, \alpha, \beta, \gamma$  and  $\eta$  is performed.

Departure process in a finite-buffer model of a bottle neck (single-server queue) subject to breakdowns (however, without setup and closedown times) was studied in details in (Kempa et. all, 2014). Applying the paradigm of embedded Markov chain, the system of integral equations for the probability distribution of the number of jobs completely processed up to fixed time  $t$ , conditioned by the initial buffer state, namely for

$$H_n(t, m) = \Pr\{h(t) = m \mid X(t) = n\}, \quad (1)$$

$$m \geq 0, \quad 0 \leq n \leq N,$$

was built, where  $X(t)$  denotes the number of jobs present in the system at time  $t$ , and  $N$  stands for the maximal system capacity (buffer size plus one). Applying the notation introduced above, the solution of the corresponding system written for mixed double transforms

$$\tilde{h}_0(s, z) = \frac{\lambda}{\lambda + s} \tilde{h}_1(s, z) + \frac{1}{\lambda + s}; \quad (2)$$

$$\tilde{h}_n(s, z) = (\lambda + \mu + \gamma + s)^{-1} \left[ \lambda \tilde{h}_{n+1}(s, z) + z\mu \tilde{h}_{n-1}(s, z) + \gamma \eta \left( \sum_{k=0}^{N-n-1} \frac{\lambda^k}{(\lambda + \eta + s)^{k+1}} \tilde{h}_{n+k}(s, z) \right) + \frac{\lambda^{N-n}}{(\eta + s)(\lambda + \eta + s)^{N-n}} \tilde{h}_N(s, z) \right] + \frac{\gamma + \eta + s}{\eta + s}, \quad 1 \leq n \leq N - 1;$$

and

$$\tilde{h}_N(s, z) \left( \mu + \gamma + s - \frac{\gamma \eta}{\eta + s} \right) = z\mu \tilde{h}_{N-1}(s, z) + \frac{\gamma + \eta + s}{\eta + s}. \quad (4)$$

was found using linear algebraic approach, where:

$$\tilde{h}_n(s, z) = \sum_{m=0}^{\infty} z^m \int_0^{\infty} e^{-st} H_n(t, m) dt, \quad (5)$$

$$|z| < 1, \quad \text{Re}(s) > 0, \quad 0 \leq n \leq N.$$

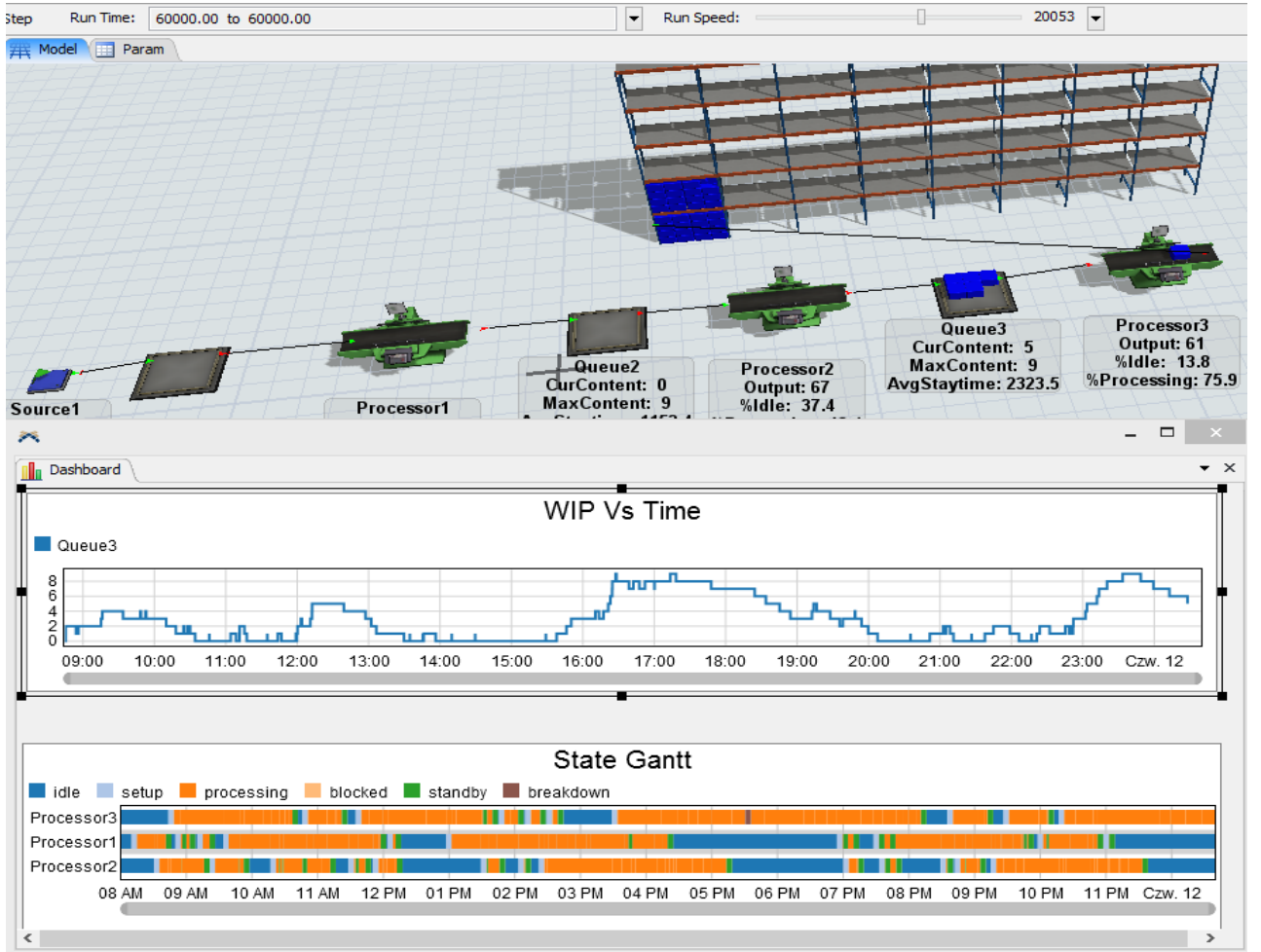


Fig. 3. The DBR model and results of the simulation run for the exemplary input data of  $\lambda, \mu, \alpha, \beta, \gamma, \eta$

Indeed (Kempa et. al, 2014), the probability generating function  $\tilde{h}_n(s, z)$  of the Laplace transform (the double mixed transform) of conditional distribution of the number of tasks completely processed in  $(0, t)$  in the finite-buffer bottleneck model described by exponential distributions, has the following form:

$$\tilde{h}_n(s, z) = \sum_{m=0}^{\infty} z^m \int_0^{\infty} e^{-st} \Pr \left\{ h(t) = m \mid X(0) = n \right\} dt =$$

$$\tilde{h}_N(s, z) \begin{bmatrix} (z\mu)^{-1} \alpha_0(s, z) \sigma(s) R_{N-n}(s, z) \\ - \sum_{k=1}^{N-n} R_{N-n-k}(s, z) \theta_k(s) \\ - (z\mu)^{-1} \alpha_0(s, z) \varpi(s) R_{N-n}(s, z) - \rho(s) \sum_{k=1}^{N-n} R_{N-n-k}(s, z), \end{bmatrix} \quad (6)$$

where  $|z| < 1$ ,  $\text{Re}(s) > 0$ ,  $0 \leq n \leq N-1$ ,

$$\text{and } \tilde{h}_N(s, z) = \frac{N_1(s, z) - N_2(s, z)}{D_1(s, z) - D_2(s, z)}.$$

The functions on the right side of (6) are defined as follows:

$$\alpha_0(s, z) = \frac{z\mu}{\lambda + \mu + \gamma + s},$$

$$\alpha_{n+1}(s, z) = \frac{1}{\lambda + \mu + \gamma + s} \left[ \frac{\gamma\eta\lambda^n}{(\lambda + \eta + s)^{n+1}} + \delta_{n,1}\lambda \right], \quad (7)$$

$$n \geq 0; \quad (17)$$

$$\theta_k(s) = \frac{\gamma\eta\lambda^k}{(\eta + s)(\lambda + \eta + s)^k (\lambda + \mu + \gamma + s)}, \quad (8)$$

$$\rho(s) = \frac{\gamma + \eta + s}{(\eta + s)(\lambda + \mu + \gamma + s)},$$

$$R_0(s, z) = 0, \quad R_1(s, z) = \alpha_0^{-1}(s, z),$$

$$R_{k+1}(s, z) = R_1(s, z) \left( R_k(s, z) - \sum_{i=0}^k \alpha_{i+1}(s, z) R_{k-i}(s, z) \right), \quad (9)$$

$k \geq 1$ .

$$\sigma(s) = \frac{(\eta + s)(\mu + \gamma + s) - \gamma\eta}{\eta + s}, \quad (10)$$

$$\varpi(s) = \frac{\gamma + \eta + s}{\eta + s}.$$

$$N_1(s, z) = (z\mu)^{-1} \alpha_0(s, z) \varpi(s) R_N(s, z) + \rho(s) \sum_{k=1}^N R_{N-k}(s, z) + (\lambda + s)^{-1} \quad (11)$$

$$N_2(s, z) = \lambda(\lambda + s)^{-1} \left[ \begin{array}{l} (z\mu)^{-1} \alpha_0(s, z) \varpi(s) R_{N-1}(s, z) + \\ \rho(s) \sum_{k=1}^{N-1} R_{N-1-k}(s, z) \end{array} \right] \quad (12)$$

$$D_1(s, z) = (z\mu)^{-1} a_0(s, z) \sigma(s) R_N(s, z) + \frac{\lambda}{\lambda + s} \sum_{k=1}^{N-1} R_{N-1-k}(s, z) \theta_k(s); \quad (13)$$

$$D_2(s, z) = \lambda((\lambda + s)z\mu)^{-1} \alpha_0(s, z) \sigma(s) R_{N-1}(s, z) + \sum_{k=1}^N R_{N-k}(s, z) \theta_k(s). \quad (14)$$

### 3. THE DBR SYSTEM MODEL

In the paper, we deal with the DBR system, in which the arrival stream of tasks is described by the uniform process with intensity  $\lambda$ . Processing tasks are executed individually by machines with service speeds described by exponential distributions with  $\mu_1 = 8$ ,  $\mu_2 = 8$  and changing  $\mu_3$ . Mean set-up time of the first and second machine equal  $\alpha_1 = 5$  and  $\alpha_2 = 5$ . Mean close-down times of the first and second machine equal  $\beta_1 = 5$  and  $\beta_2 = 5$ . Set-up times of the bottleneck are described by the exponential distribution with  $\alpha_3$ . Close-down time of the bottleneck is equal  $\beta_3$ . The total DBR system capacity equals  $N$ . If the task arriving to the system finds the time buffer occupied, it is lost. Arriving tasks from a single waiting line and are served in the order of their arrivals. At time  $t = 0$  the first failure-free period of the bottleneck begins which ends with a failure after an exponentially distributed time with mean  $\gamma = 500$ . Successively, the repair time of the bottleneck with mean  $\eta = 16$  begins immediately at the end of failure-free time.

The DBR system is modeled and simulations are run in the Flexsim Software. The DBR model is presented in Figure 3. Results of the simulation run for the exemplary input data of  $\lambda$ ,  $\mu$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\eta$  and  $n$ . Resulting WIP for the constraint buffer is presented in the queue diagram, and working status of the bottleneck is presented in the Gantt chart (Figure

3). These characteristics were achieved after  $T=1000$ min, taking mean value of interarrival time equal to 10min. Mean processing, set-up, close-down times are equal to 13, 5 and 5min, respectively, for the bottleneck. The level of the constraint buffer saturation  $n = 4$ . Whenever the constraint buffer is empty, the bottleneck is stopped by a close-down time. The bottleneck needs a set-up time before processing of the first task after the idle time.

### 4. THE SIMULATION RESULTS

Below we study in details the influence of the key parameters  $\lambda$ ,  $\mu$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\eta$  and  $n$  on the mean TR in different scenarios of the DBR system operation.

#### 4.1 Effect of the intensity of arrivals on throughput of DBR system

Investigate, firstly, the effect of  $\lambda$  on the number of tasks completely processed (TR of the DBR system) before the fixed time epoch  $T$ . Let us observe this characteristic after  $T=1000$ min, taking mean values of  $\mu$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\eta$  presented in Section 3. For 9 different values of  $\lambda$  of two successive tasks, namely 10, 11, ..., 18min, we compare the mean TR before  $T$  for level of the constraint buffer saturation:  $n = 4$ . Number of replicates per one set of input data is 15. The results of simulations are presented in Figure 4. As one can observe, as  $\lambda$  increases (the arrival intensity decreases), the mean TR decreases. In addition, it should be noted that for lower  $\lambda$  values, the average TR remains stable during the period in which the bottleneck starts the rope, sending information to the buffer to limit the number of products before the bottleneck (WIP). After increasing the interarrival time, the number of orders entering the system does not meet the bottleneck demand, and the productivity decreases.

#### 4.2 Effect of speed of processing of the bottleneck of the DBR system

For 10 different values of  $\mu_3$  of bottleneck, namely 17, 16, ..., 8min, we compare the mean TR before  $T$  for level of the constraint buffer saturation:  $n = 4$ . Number of replicates per one set of input data is 15. The results of simulations are presented in Figure 5. As can be seen, as the bottleneck processing time decreases (the processing speed is increased), the average TR increases. This happens until  $M_3$  ceases to be a bottleneck.

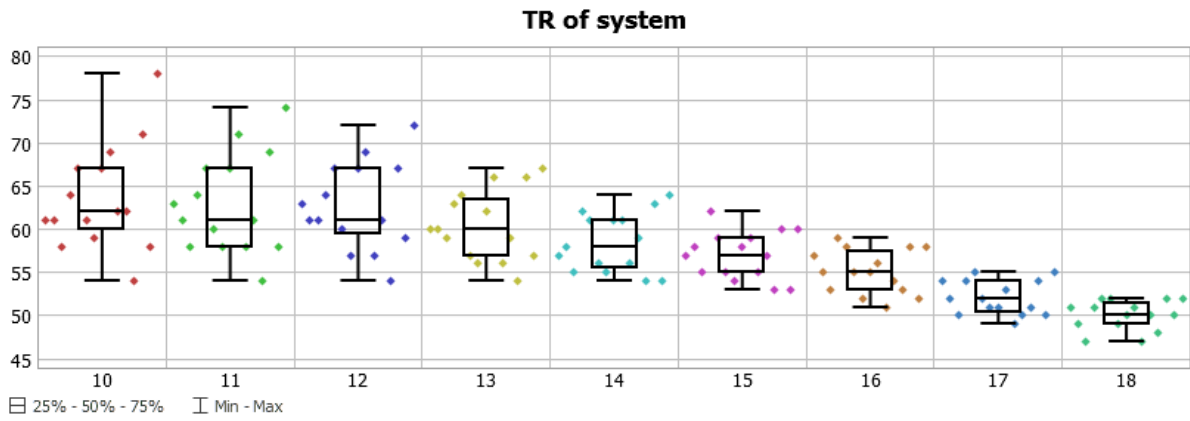


Fig. 4. TR as a function of  $\lambda = 10, 11, \dots, 18$  after  $T=1000$ min for  $\mu_{1,2}=8, \mu_3=13, \alpha_{1,2,3}=\beta_{1,2,3}=5, \gamma = 500, \eta = 16$  and  $n = 4$

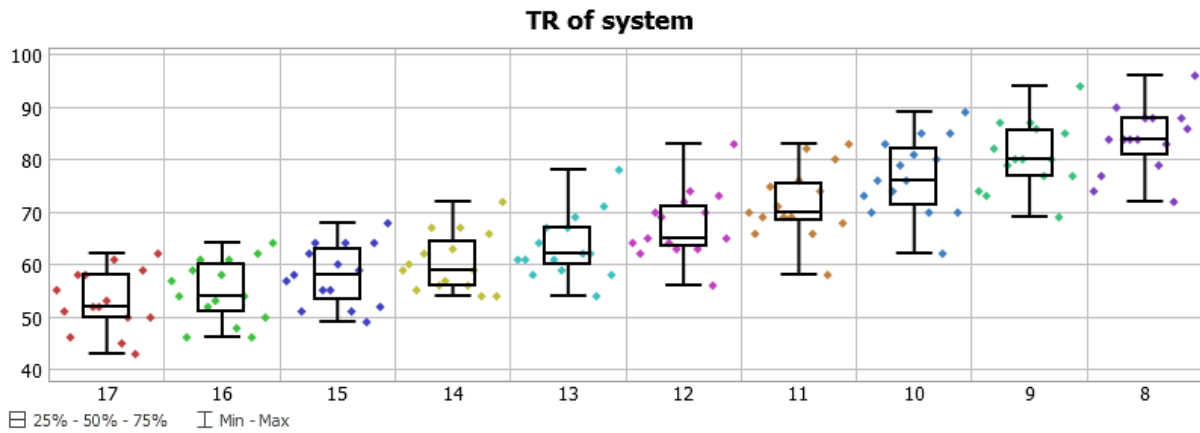


Fig. 5. TR as a function of  $\mu_3 = 17 \dots 8$  after  $T=1000$ min for  $\mu_{1,2}=8, \lambda = 10, \alpha_{1,2,3}=\beta_{1,2,3}=5, \gamma = 500, \eta = 16$  and  $n = 4$ .

#### 4.3 Effect of the failure-free time of the bottleneck on throughput of the DBR system

For 10 different values of  $\gamma = 500, 450, \dots, 50$ min, we compare the mean TR before  $T$  for level of the constraint buffer saturation:  $n=4$ . Number of replicates

per one set of input data is 15. The results of simulations are presented in Figure 6. As one can see, the value of  $\gamma$  does not affect the system's performance until the value of 200min is exceeded, so it approaches the value of processing times.

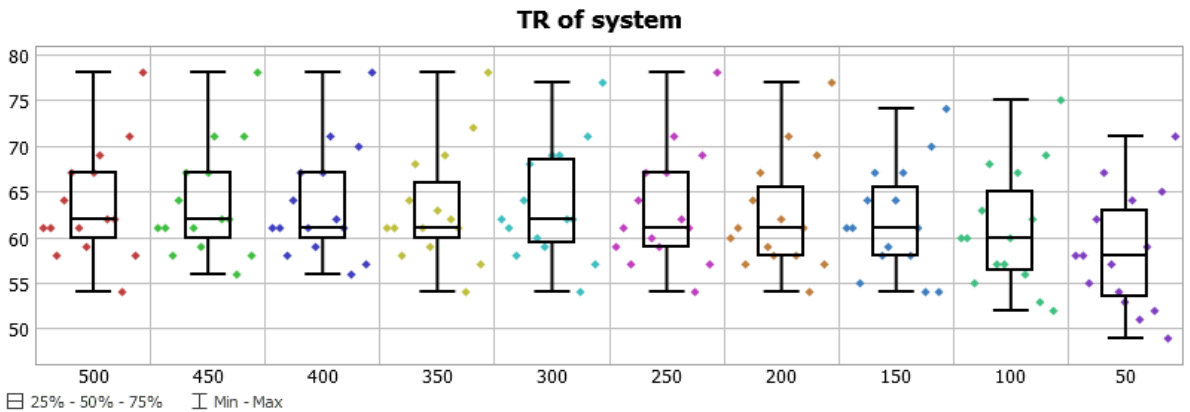


Fig. 6. TR as a function of  $\gamma = 500, 450, \dots, 50$  after  $T=1000$ min for  $\mu_{1,2}=8, \mu_3=13, \alpha_{1,2,3}=\beta_{1,2,3}=5, \lambda = 10, \eta = 16$  and  $n = 4$ .

#### 4.4 Impact of the level of the constraint buffer saturation on throughput of the DBR system

For 9 different values of  $n=0, 1, \dots, 8$ , we compare the mean TR before  $T$  for level of the constraint buffer saturation:  $n=4$ . Number of replicates per one set of input data is 15. The results of simulations are presented in Figure 7. In this case, a significant impact on system

performance can be observed for values from 0 to 5. For these values, the time buffer is insufficient to protect the bottleneck against processing time variations and system disturbances. Above these values, the buffer size is sufficient and does not further increase system performance.

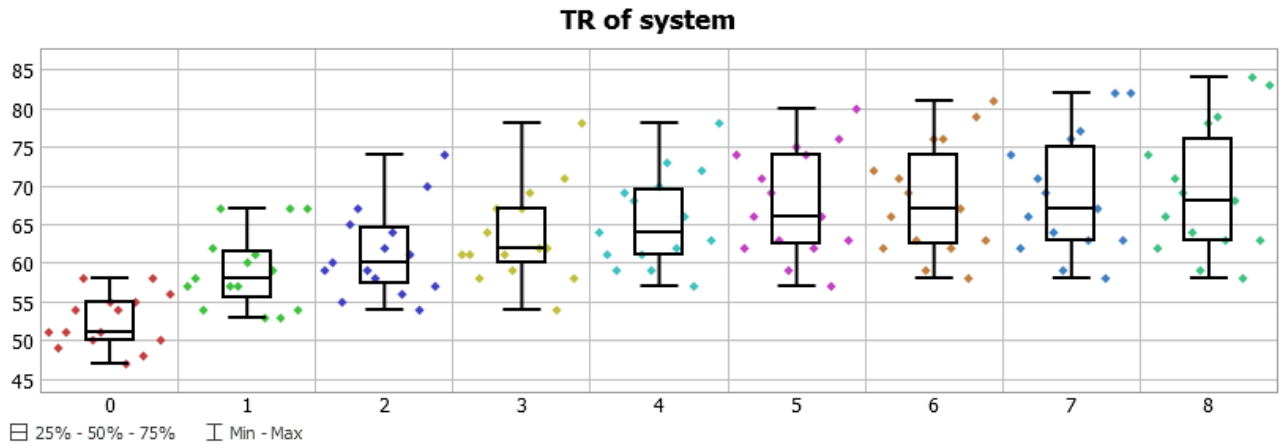


Fig. 7. TR as a function of  $n=0,1,\dots,8$  after  $T=1000\text{min}$  for  $\mu_{1,2}=8, \mu_3=13, \alpha_{1,2,3}=\beta_{1,2,3}=5, \lambda=10, \gamma=500, \eta=16$ .

#### 4.5 Impact of the setup time on throughput of the DBR system

For 10 different values of  $\alpha_{1,2,3} = 1, \dots, 10$ , we compare the mean TR before  $T$  for level of the constraint buffer saturation:  $n = 4$ . Number of replicates per one set of

input data is 15. The results of simulations are presented in Figure 8. In this case, a proportional decrease in performance can be observed with increasing setup times and their relation to the processing time.

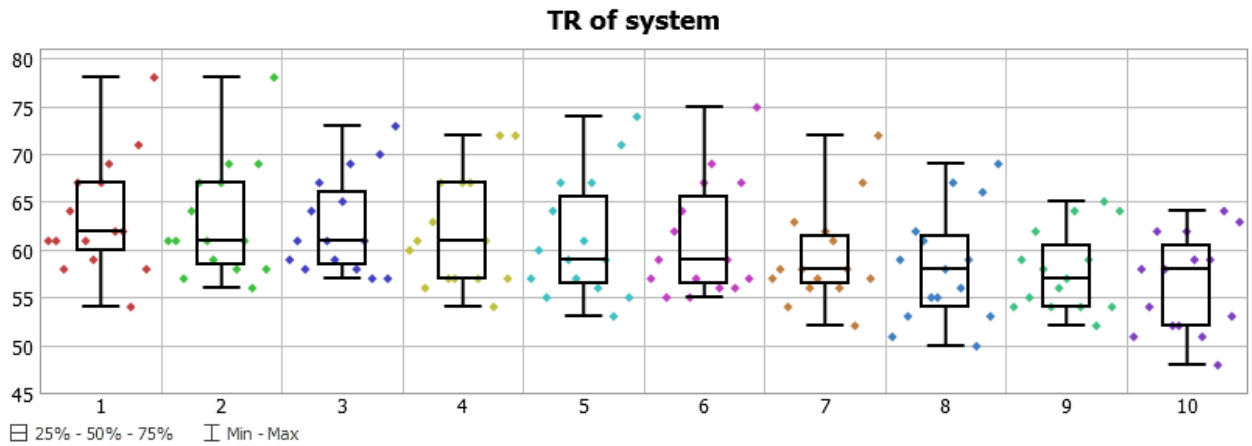


Fig. 8. TR as a function of  $\alpha_{1,2,3}=1, 2, \dots, 10$  after  $T=1000\text{min}$  for  $\mu_{1,2}=8, \mu_3=13, \beta_{1,2,3}=5, \gamma=500, \lambda=10, \eta=16$  and  $n=4$ .

#### 4.6 Impact of the closedown time on throughput of the DBR system

For 10 different values of  $\beta_{1,2,3} = 0, 1, \dots, 9$ , we compare the mean TR before  $T$  for level of the

constraint buffer saturation:  $n=4$ . Number of replicates per one set of input data is 15. The results of simulations are presented in Figure 9. For given values of  $\beta$  there is little relation between TR and  $\beta$ .

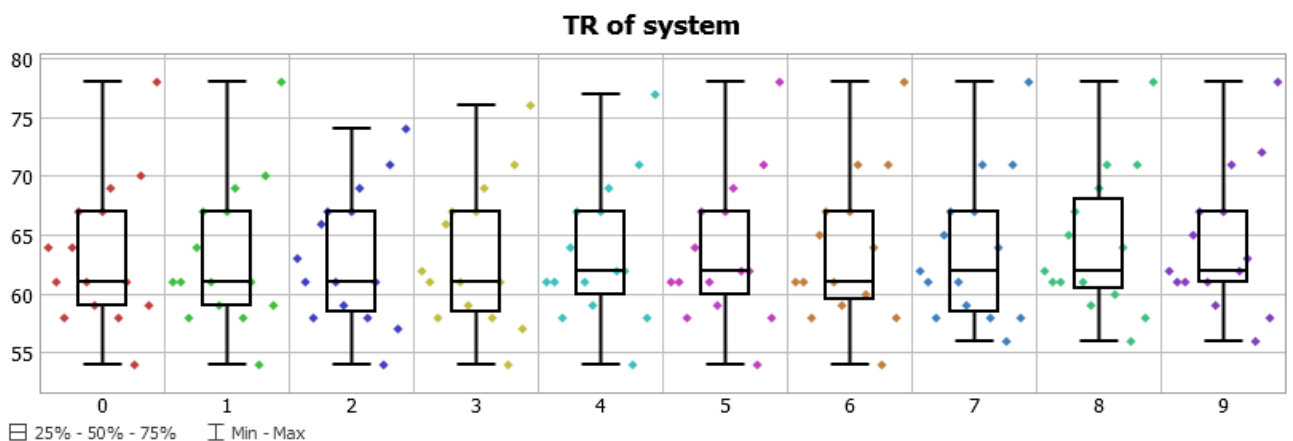


Fig. 9. TR as a function of  $\beta_{1,2,3}=1, 2, \dots, 10$  after  $T=1000\text{min}$  for  $\mu_{1,2}=8, \mu_3=13, \alpha_{1,2,3}=5, \gamma=500, \lambda=10, \eta=16$  and  $n=4$ .



## 5. CONCLUSIONS

In the paper the impact of “input” DBR system parameters, namely the intensity of arrivals, speed of processing, speed of setup and closedown times of the bottleneck, failure-free period duration and repair time on the mean TR occurring in the fixed time horizon  $[0, T)$  was investigated. The parameters that have the greatest impact on the performance of the DBR system are: intensity of arrivals and speed of processing of the bottleneck. Also, the constraint buffer saturation has a significant impact on the DBR system performance. It is important to know a sufficient size of the constraint buffer to protect the bottleneck against processing time variations and system disturbances. Analysed values of set-up and close-down times had little effect on the throughput of the DBR system.

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