

## SYNTHESIS OF TOOTH-LEVER DIFFERENTIAL GEAR MECHANISM

Gayrat Bahadirov, Abdusalam Abdukarimov

Institute of Mechanics and Seismic Stability of Structures, Uzbek Academy of Sciences, Acad. Jahyo Guljamova, No. 70,  
100047 Tashkent, Uzbekistan

Corresponding author: Gayrat Bahadirov, instmech@rambler.ru

**Abstract:** Solution of the problem of synthesis of inter-roller gear mechanism is considered in the paper. Four-wheel tooth-lever gear mechanism of differential type with two-contour lever four-bar link was studied, where the first contour is a slider-crank and the second one – lever parallelogram. Gear mechanism of such type may be applied in two-roller modulus to transfer torsion moment from one working roll to another one, where the constancy of transmission relation at the time of change of interaxial distance between working rollers is necessary. The aim of this work is to determine geometrical parameters of gear mechanism depending on geometrical parameters of two-roller modulus with minimal and maximal interaxial distances at the time of operation of technological process and at the time of repairing works, as well as depending on torsional moment transferred by tooth wheels with account of dynamic factors, such as the angles of pressure of lever contours. Development of the method of synthesis of tooth-lever differential mechanism of gear in application to two-roller modulus with variable interaxial distances of working rollers also presents one of the goals of the paper.

**Key words:** Synthesis, tooth-lever mechanism, angle of pressure, differential, working roller.

### 1. INTRODUCTION

In technological process interaxial distance of working rollers is changing in certain roller technological machines. Inter-roller gear mechanism of such roller machines, which transfers torsional moment from one working roll to another one, should satisfy (among others) one principal condition, such as to ensure rotation of working rollers with similar in value and direction of linear velocities of the point of contact of these rollers and processed material both with constant and changing interaxial distances of working rollers.

However, gear mechanisms of certain technological machines, applied in different fields of economy, do not fully satisfy this requirement.

For example, tooth-lever mechanism of gear of machines for processing the stalks of bast and kenaf (Kuznetsov and Smirnov 1967), tooth-lever mechanism of gear in cotton harvesting machine, tooth mechanism of gear, applied in wringing machine VOPM-1800-K, tooth-lever mechanism of gear in staking - cloth mellowing machine TMPH-

1800-K, chain mechanism of gear of wringing machine made by “Svit” company (Burmistrov, 2006), which are aimed to transfer torsional moment from one working roller to another one with constant gear number, could ensure constancy of gear relation only at constant interaxial distance between working rollers; at the moment of change of interaxial distance of working rollers, gear relation is also changing; that leads to the failure of fulfillment of agro-technical and technological requirements to machines. Such failures may finally lead to worsening of the quality of processed material, sometimes to its damage, decrease in efficiency and durability of machine operation (Kuznetsov and Smirnov, 1967).

There are many such examples. Such demerits in design were due to the lack of studies of roller technological machines and their operating mechanisms on the whole, and inter-roller gear mechanisms in particular, as well as due to the lack of the methods of structural, kinematic, dynamic analysis and synthesis of these gear mechanisms.

We have worked out (Abdukharimov et al., 1990) and studied (Bahadirov, 2010) gear mechanism, that lacks above-mentioned demerits.

Worked out by the authors gear mechanism presents tooth-lever gear mechanism of differential type. Tooth-lever mechanisms are applied in engineering for a long time. However some essential characteristics of these mechanisms were revealed only recently; they allow consider them as one of the most progressing mechanisms to create modern machines and devices (Fateev, 2009). Tooth-lever mechanisms are the mechanisms with both lowest and highest kinematic pairs, which are connected parallelly or consecutively (Volmer, 1969). A special attention was paid to the theory of tooth-lever mechanisms in Germany, in particular, to four-wheel and three-wheel tooth-lever mechanisms. For example in the works of K.Hain, W.Lichtenheldt, J.Volmer and others (Hain, 1961, Lichtenheldt, 1965). The following researchers were also interested in the theory of tooth-lever mechanisms: Shashkin A.S., Karelin V.S., Maysyuk L.B., Polukhin B.P., Beletsky V.Ya., Ambartsumyantz R.V., Zakirov G.Sh., Fateev N.A., and many others (Levitsky,

Shashkin A.S. has described the method of the synthesis of tooth-lever mechanism, in which under uniform rotation of a crank exit link performs one-way motion with approximate stop of a given duration. Characteristics of centroid in relative motion of links were used. He has drawn a formula, that permits to equate sought for parameters of the synthesis from conditions of approximation of centroid to the arc of circumference (Shashkin, 1987). Maysyuk L.B., Polukhin V.P. on the example of the simplest tooth-lever mechanisms have described the method of the synthesis of universal tooth-lever mechanisms able to carry out the functions of regulators in self-adjusting devices. The method is based on the regularities of kinematic geometry. In a special case of location of instantaneous poles and a pole of finite turning of location of plane figure the curve of centers falls apart on circumference and a straight line. When this circumference coincides with initial circumference of a driven wheel of mechanism, the effect of regulated on the passage one-way periodical motion is achieved (Maysyuk and Polukhin, 1974).

Zakirov G.Sh, Zakirova M.Sh., Khasanov I. have studied the problem of kinematic synthesis on PC of tooth-lever mechanism with reversing satellites. They have determined the values of parameters of mechanism, when at one rotation of a crank, a satellite performed six rotations clockwise, then two rotations anticlockwise, that is the motion of a reverse of satellite (Zakirov et al., 1994).

An analysis of works on tooth-lever mechanisms shows that, in principal, the authors considered the synthesis of tooth-lever mechanisms, in which periodical rotational motion of entrance link was transformed into periodical rotational motion of exit link with stoppage, or periodical rotational motion of entrance link was transformed into rotational motion of exit link with variable velocity.

We consider tooth-lever mechanism, in which rotational motion of entrance link is transformed into synchronic rotational motion in opposite direction, but it is not changed in value with the change of interaxial distance between drive tooth wheels and driven ones.

Weak development of the methods of analysis and synthesis of tooth-lever mechanisms, especially when applying these mechanisms as inter-roller gear mechanisms for roller moduluses, were pointed out by many authors such as N.I.Levitskiy, K.V.Frolov, V.S.Karelin and others (Levitskaya and Levitsky, 1985; Frolov et al., 2001; Karelin, 1986).

Elaboration of new structures of tooth-lever mechanisms for roller machines of different structures, development of the methods of synthesis of tooth-lever mechanisms for the schemes of roller machines with arc-type motion of the center of

rotation of a driven tooth wheel and creation of classification of roller machines and gear mechanisms in application to these roller machines present the aim of our reseach in future.

## 2. SYNTHESIS OF A MECHANISM

Fig.1. shows design scheme of the synthesis of worked out tooth-lever differential gear mechanism.

From kinematic and dynamic analysis which was made by us before, of this mechanism, it can be seen that, for satisfying the main condition, at projecting two-roller modulus with similar diameters of working rollers, it is necessary, that (Abdukarimov et al., 2004):

1. Tooth contour of tooth-lever mechanism should consist of four tooth wheels, if tooth wheels have external gearing.
2. The number of teeth of tooth wheels should be the same in all wheels or pairwise similar, two parasite and driven with drive ones.
3. Lever contour of tooth-lever mechanism should be parallelogram.
4. Rocker-and-slider contour should be axial.
5. The mechanism should ensure the change of interaxial distance of working rollers at performing technological process on a value  $W_1$ , and at repairing works in roller machine on a value  $W_2$ .
6. At the performance of technological process the angles of pressure between lever links must be less than permissible ones.
7. Diameters of circumferences of tooth points of driven and drive tooth wheels should be less than minimal diameter of working rollers on a guaranteed clearance between tooth points of tooth wheels.

Let us assume that it is necessary to design roller machine with diameters of working rollers  $D_{\theta_1}$  and

$D_{\theta_2}$  with the change of interaxial distance in the

process of work on a value  $W_1$ , in the process of repairing works on a value  $W_2$ . On the basis of

above given conditions we may write down:

$$D_{\theta} = D_{\theta_1} = D_{\theta_2} \quad (1)$$

$$AD_{p.\max} = D_{\theta} + W_1 \quad (2)$$

$$AD_{p.\min} = D_{\theta} \quad (3)$$

$$AD_{n.\max} = D_{\theta} + W_2 \quad (4)$$

$$AD_{n.\min} = D_{\theta} \quad (5)$$

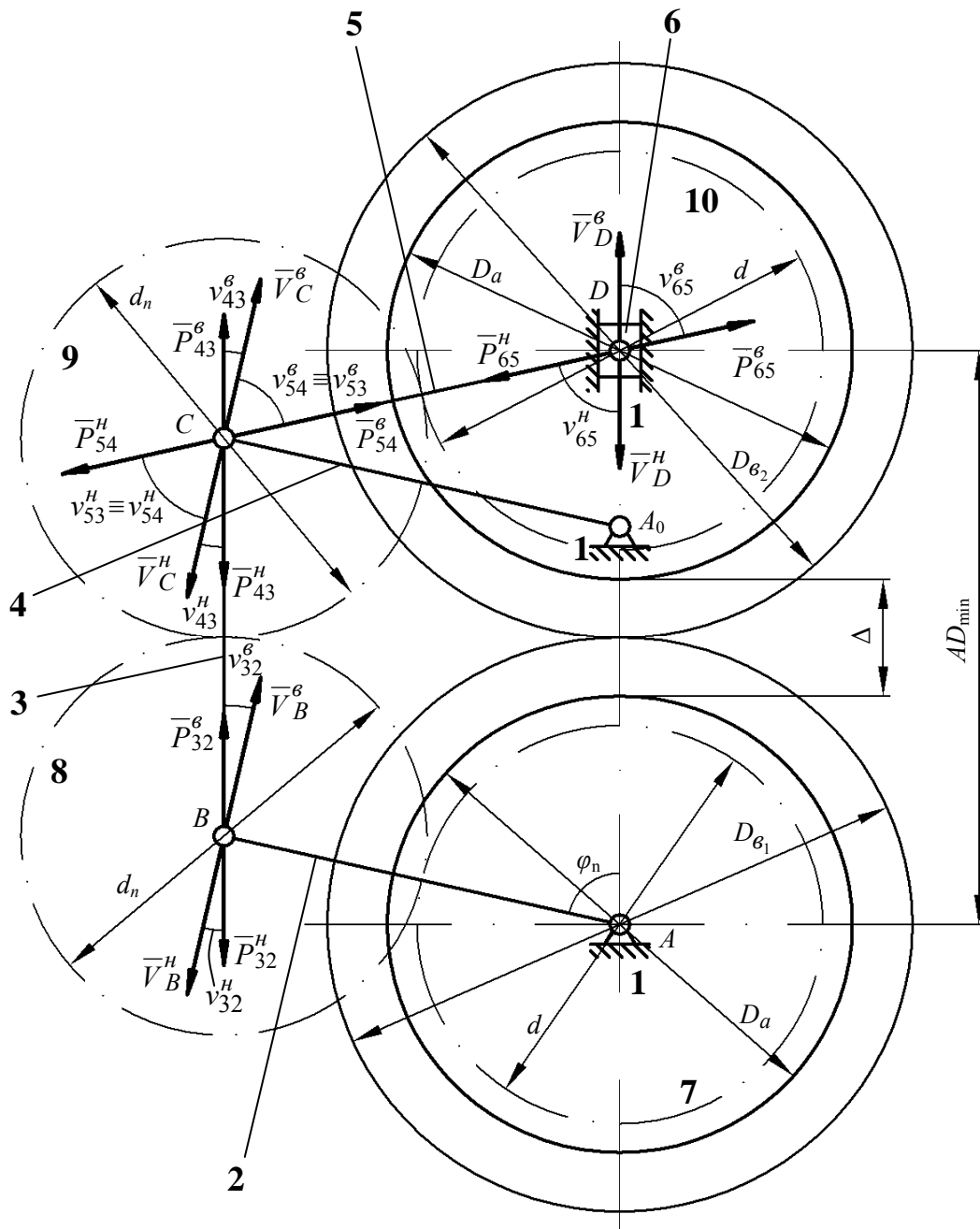


Fig. 1. Design scheme of synthesis of tooth-lever gear mechanism with parallelogram lever contour. 1 – fixed post, 2, 3, 4, 5 – levers, 6 – slider, 7 – drive tooth wheel, 8, 9 – intermediate tooth wheels, 10 – driven tooth wheel

$$D_a = D_{a_1} = D_{a_2} \quad (6)$$

$$D_a = D_{\delta} - \Delta \quad (7)$$

Here  $D_{\delta_1}$  and  $D_{\delta_2}$  - are diameters of driven and drive working rollers, respectively;

$AD_{p.max}$  and  $AD_{p.min}$  - maximal and minimal interaxial distance of working rollers at performing of technological process;

$AD_{n.max}$  and  $AD_{n.min}$  - maximal and minimal interaxial distances of working rollers at repairing works;

$D_{a_1}$  and  $D_{a_2}$  - diameters of circumferences of tooth points of driven and drive tooth wheels;

$\Delta$  – guaranteed clearance between tooth points of driven and drive tooth wheels.

The emphasis on tooth points of drive and driven tooth wheels at the beginning of projecting is due to the fact that, under the change of interaxial distance of working rollers, the levers supporting intermediate wheels are subjected to the load from momentum force, depending on mass and acceleration of the centers of rotation of these tooth wheels. Hence at large acceleration of the change in interaxial distance, it is advisable to acquire geometrical parameters of drive and driven tooth wheels as maximal ones, and

diameters of intermediate tooth wheels – as minimal ones.

Knowing the largest moment transmitted by tooth wheels, we may determine preliminary interaxial distance of tooth wheels ( $a_p$ ), preliminary width of tooth wheels ( $\epsilon_p$ ) and preliminary modulus of a tooth ( $m_p$ ), assuming that driven and drive tooth wheels are in gearing (Kolesnikov, 1999).

As diameters of tooth points of driven and drive tooth wheels are given, we may determine preliminary dividing diameter of these tooth wheels

$$d_p = (D_e - \Delta) - 2m_p \quad (8)$$

As well as preliminary number of teeth

$$Z_p = \frac{d_p}{m_p} \quad (9)$$

We round up the number of teeth  $Z_p$  up to an integer  $Z$  in decreasing way and determine preliminary modulus of a tooth

$$m'_p = \frac{d_p}{Z} \quad (10)$$

According to the table of moduluses we obtain final modulus of a tooth ( $m$ ).

Further we determine final dividing diameter

$$d = mZ \quad (11)$$

final diameter of tooth points and final guaranteed clearance

$$D_a = d + 2m \quad (12)$$

$$\Delta = D_e - D_a \quad (13)$$

Consider lever contours of tooth-lever mechanism.

In design of a mechanism it is necessary to account a very important parameter, which characterizes the condition of force transmission and operability of the mechanism, namely an angle of pressure  $\nu$ . Maximal value of an angle of pressure should not exceed permissible value, that is  $\nu_{\max} \leq [\nu_{adm}]$ .

In discussed mechanism (Fig.1) lever part of gear mechanism consists of two contours:

- 1) rocker-and-slider contour;
- 2) lever four-bar link (parallelogram).

In the first contour slider 6 is a drive link, rocker 4 – a driven link. At the passage of slider 6 from drive

tooth when an angle of pressure in kinematic pair  $D$  is an angle  $\nu_{65}^g$  (an angle between the force  $\overline{P}_{65}^g$  directed on the link 5 from the link 6 and vector of velocity  $\overline{V}_D^g$ , directed along the passage of a slider 6). At the passage of a slider 6 to drive tooth wheel, an angle of pressure  $\nu_{65}^H$  - is an angle between the force  $\overline{P}_{65}^H$ , directed on the link 5 from the link 6 and vector of velocity  $\overline{V}_D^H$  directed along the passage of slider 6. Between the links 5 and 4 in kinematic pair C angles of pressure are  $\nu_{54}^g$  and  $\nu_{54}^H$  respectively. Angles of pressure between links 5 and 3 in kinematic pair C  $\nu_{53}^g$  and  $\nu_{53}^H$  fully coincide with the angles of pressure between links 5 and 4.

$$\nu_{53}^g = \nu_{53}^H = \nu_{53} = \nu_{54} \quad (14)$$

From design scheme it is seen that

$$\nu_{65}^g = \nu_{65}^H = \nu_{65} = \varphi_n \quad (15)$$

$$\nu_{54}^g = \nu_{54}^H = \nu_{54} = 2\varphi_n - 90^\circ \quad (16)$$

Hence an angle of location of mechanism is determined as

$$\varphi_n = \frac{\nu_{54} + 90^\circ}{2} \quad (17)$$

In the second contour drive link is a lever 4, so the angles of pressure in kinematic pair C between the links 4 and 3 are  $\nu_{43}^g$  and  $\nu_{43}^H$ , in kinematic pair B between the links 3 and 2 are  $\nu_{32}^g$  and  $\nu_{32}^H$ . From design scheme it is seen that:

$$\nu_{43}^g = \nu_{43}^H = \nu_{43} \quad (18)$$

$$\nu_{32}^g = \nu_{32}^H = \nu_{32} \quad (19)$$

$$\nu_{32} = \nu_{43} \quad (20)$$

$$\nu_{32} = 90^\circ - \varphi_n \quad (21)$$

So we may write down

$$\varphi_n = 90^\circ - \nu_{32} \quad (22)$$

Equalling the right sides of formulae (17) and (22) we may write down

$$v_{54} = 90^\circ - 2v_{32} \quad (23)$$

From formulae (15), (17), (22) it is seen that the increase of the angle  $\varphi_n$  leads to the decrease of an angle of pressure  $v_{32}$  from one side, and to the increase of the angles of pressure  $v_{54}$  and  $v_{65}$  from another side. In preliminary design for the mechanisms with rotational pairs only it is taken as  $[v] = 45^\circ - 60^\circ$ , with the presence of translational kinematic pairs  $[v] = 30^\circ - 45^\circ$ , so,

$$[v_{65}] = 45^\circ - 60^\circ \quad (24)$$

$$[v_{54}] = 30^\circ - 45^\circ \quad (25)$$

$$[v_{32}] = 30^\circ - 45^\circ \quad (26)$$

Formulae (15), (16) and (21) satisfy conditions (24), (25) and (26) at  $\varphi_n = 45^\circ - 30^\circ$ .

Fig. 2 shows the graphs of changes of the angles of pressure  $v_{32}$ ,  $v_{43}$ ,  $v_{54}$ ,  $v_{65}$  depending on the angle of location of mechanism  $\varphi_n$ . Actual change of the angle of location of mechanism  $\varphi_\partial$  should be less, than  $15^\circ (45^\circ - 30^\circ)$ . It is necessary to state that the value of admissible angles of pressure  $[v_{54}] = [v_{53}]$  may be within the interval from  $(-45^\circ) \div (-30^\circ)$  till  $(+30^\circ) \div (+45^\circ)$ , that corresponds to the angle of location of mechanism from  $22,5^\circ$  to  $67,5^\circ$  according to formula (17).

Further let us consider the determination of dividing radius ( $r_n$ ) of intermediate wheel at an angle of location of mechanism  $\varphi_n = 45^\circ$  and the value of minimal angle of location of mechanism ( $\varphi_{n.min}$ ) satisfying the condition of change of interaxial distance in working position ( $W_1$ ).

From design scheme (Fig. 1) it may be written

$$AD_{p.min} = 2(r + r_n) \cos \varphi_{n.max} + 2r_n \quad (27)$$

$$AD_{p.max} = 2(r + r_n) \cos \varphi_{n.min} + 2r_n \quad (28)$$

$$AD_{p.min} = d + \Delta, \quad (29)$$

$$AD_{p.max} - AD_{p.min} = W_1 \quad (30)$$

here  $r$  – is a dividing radius of drive and driven tooth wheels.

From formula (27) with account of (28), (29), (30) one obtains

$$r_n = \frac{d + \Delta - r\sqrt{2}}{2 + \sqrt{2}} \quad (31)$$

here  $r_n$  - dividing radius of intermediate wheels, ensuring  $D_a + \Delta = AD_{p.min}$  at an angle of location of mechanism  $\varphi_n = 45^\circ$ .

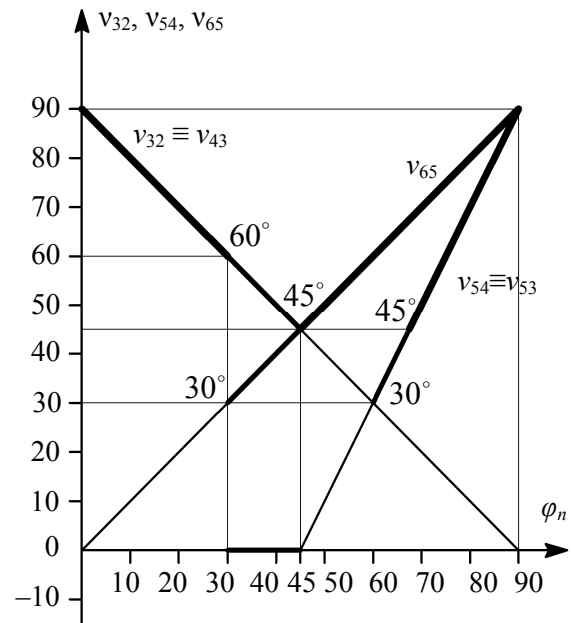


Fig. 2. Graphs of changes of the angles of pressure  $v_{32}$ ,  $v_{43}$ ,  $v_{54}$ ,  $v_{65}$  depending on the angle of location of mechanism  $\varphi_n$

Subtracting (27) from formula (28) and taking into consideration formulae (30) and (31) one may write down

$$\varphi_{n.min} = \arccos \left( \frac{W_1(2 + \sqrt{2}) + \sqrt{2}(2d + \Delta)}{2(2d + \Delta)} \right) \quad (32)$$

Substituting into formula (32) given value of  $W_1$  and value of  $d$  from formula (11) we determine  $\varphi_{n.min}$ .

If  $\varphi_{n.min} \geq [\varphi_{n.min} = 30^\circ]$ , we will perform further calculations, if  $\varphi_{n.min} < [\varphi_{n.min} = 30^\circ]$ , increasing  $Z_n$  on one tooth, we determine  $\varphi_{n.min}$  anew, till satisfying the condition

$$\varphi_{n.min} \geq [\varphi_{n.min} = 30^\circ].$$

After satisfying the condition and obtaining calculated value of  $Z_n$ , we determine final dividing diameter of intermediate wheels

$$d_n = Z_n \cdot m \quad (33)$$

Then the length of lever and maximal value of inter-roller distance are determined

$$l_{AB} = l_{CD} = \frac{d + d_n}{2} \quad (34)$$

$$l_{CB} = l_{AA_0} = d_n \quad (35)$$

$$AD_{n.max} = D_\delta + W_2 \quad (36)$$

$$W_2 = d + 2d_n - D \quad (37)$$

In projecting of gear mechanism for roller machines with relatively small acceleration of the center of rotation of driven working roller, it is advisory to take dividing diameters of all tooth wheels as equal to  $d = d_n$ , with account of geometrical parameters of roller pair and technological requirements to them; this is rational both from economic and exploitational points of view.

### 3. CONCLUSIONS

The method of synthesis of tooth-lever differential mechanism of gear was worked out in application to two-roller moduluses with variable interaxial distance of working rollers.

Formulae to determine the angle of location of a mechanism, depending on the angles of pressure were formulated, the values of the angles of location of the links corresponding to admissible values of the angles of pressure were determined.

Formulae to determine geometrical parameters of the mechanism of gear, depending on geometrical parameters of two-roller modulus and technological requirements to modulus, which are expressed by the value of maximal change of inter-roller distance at the time of performance of technological process and at the time of repairing works, as well as depending on transmitted by tooth wheels torsional moment with account of dynamic factors, such as rational angles of pressure of lever contours were drawn.

### 4. REFERENCES

1. Kuznetsov G.K., Smirnov B.I. (1967). *Projecting of machines of primary processing of bast fiber*, pp. 59-63, Mashinostroenie, Moscow.
2. Burmistrov A.G., (2006). *Machines and apparatus of leather and fur production*, pp. 34-40, ISBN 5-

9532-0284-9, KolosS, Moscow.

3. Amanov T.Yu., Abdurkarimov A., Bahadirov G.A., (1990). *Gear mechanism of roller machines*, Patent 1632047 A 1 USSR, MKI C14 B 1/00, 1/02.. /- No. 4621476/12; Appl. 19.12.88.
4. Bahadirov, G.A. (2010). *Mechanics of wringing roller pair*, pp. 113-137, ISBN 978-9943-19-018-4, Fan, Tashkent.
5. Fateev N.A., (2009). *Determination of conditional velocities and angular accelerations of the links of plane tooth-lever mechanisms*, Materials of XXXVIII Science and Engineering Conference of the work of faculty members Sev.Kav.GTU., Stavropol, Available from: www.ncstu.ru. Accessed: 05.10.2011
6. Volmer J., (1969). *Getriebetechnik*. Lehrbuch. VEB –Verlag “Technik”, Berlin.
7. Hain K. (1961). *Angewandte Getriebelehre*. VDI – Verlag, Düsseldorf.
8. Lichtenheldt W., (1965). *Konstruktionslehre der Getriebe Akademie*, Verlag, Berlin.
9. Levitsky N.I., (1974). *Theory and application of tooth-lever mechanisms*, pp. 5-8, pp. 135-136, Edited by Levitsky, Nauka, Moscow.
10. Shashkin A.S., (1987). *Regulated tooth-lever and wave mechanisms*, pp. 89-106, Voronej University Publishing House, Voronej.
11. Maysyuk L.B., Polukhin V.P., (1974). *The method of synthesis of tooth-lever mechanisms with the duration of stoppage, regulated on the path*, Theory and application of tooth-lever mechanisms. pp. 22–25, Nauka, Moscow.
12. Zakirov G.Sh., Zakirova M.Sh., Khasanov I., (1994). *Kinematic synthesis on PC of tooth-lever mechanism with reversing satellites*, Uzbek Journal Problems of mechanics, No. 3-4, pp. 29-34, Fan, Tashkent.
13. Levitskaya O.N., Levitsky N.I., (1985). *Course of the theory of mechanisms and machines*, pp. 177-178, Vysshaya shkola, Moscow.
14. Frolov K.V., Popov S.A., Musatov A.K. et al. (2001). *Theory of mechanisms and mechanics of machines*, pp. 442-444, Textbook for students, Frolov K.V. (Ed.), Vysshaya shkola, Moscow.
15. Karelin B.S., (1986). *Projecting of lever and tooth-lever mechanisms*, pp. 138-155, Mashinostroenie, Moscow.
16. Abdurkarimov A. et al., (2004). *Power analysis of the gear mechanism of roller pair of squeezing machine*, Science technological journal FerPI, No. 4, pp. 31-35.
17. Kolesnikov K.S. et al., (1999). Mashinostroenie. Part 1: *Engineering methods of design*. Volume. 1-2; *Theoretical mechanics, thermo-dynamics, heat-transfer*, Kolesnikov K.S. (Ed.).