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PACKAGE OF CALCULUS, DRAWING AND DESIGN OF THE ROLLING TOOLS FOR MANUFACTURING OF THE HELICAL SURFACES

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Abstract: Package of calculus, drawing and design of the rolling tools for the manufacturing of the helical surfaces within the structure of the helical compressors. The estimation of the hobs or of the rake tools for the manufacturing of the rotors of the helical compressors represents a highly difficult task for the tool designers. Starting with the construction of the calculus device and the conception of some methods of quick verification of the correctness of the analytical results, we performed the analysis of all the steps involved in the process of design and the automation of these procedures. Using the drawing environment AutoCAD as a supportive element, we transposed all the profiles involved in the process of calculation and we designed graphs allowing the quick and precise verification of the results obtained through analytical methods; these are then used in the construction of the tools and the simulation of the processes of interaction between the rotor and the tool. This first paper is dedicated to rack type cutter.
Key words: rolling tools; rack type cutter; methods of profiling of rack type cutter; CAD for helical surfaces; screw compressors.

1. INTRODUCTION

The rolling tools are very often used to define and to obtain the profiles of helical compressors. The performance of the screw compressor depends mainly on the tooth profile of mating rotors. In literature of screw compressors analysis of the rotors has been made by conjugation of surfaces or rotors and synthesis of rotor surfaces with two lines of contact [1]. The rack was proposed by Rinder [2] for generation of rotor profile based on gearing theory. Later Stosic [3] proposed a pair of rack-generated rotors in order to open possibilities of optimization of screw compressors. Also Stosic with his team [4, 5] treated the tools for manufacturing of screw compressors and they presented the rack type cutter. Wu [6] used a new possibility of definition of the rotors by rack. With development of the software CAD new method of obtain the rack profile is presented in Berbinschi [7].
But for design of the hob and also the rotors with helical surfaces must be created a package of calculus, drawing and every engineer use different mathematical approach and different structure of the software. In this sense the paper presents some methods for profiling the rack type cutter with the possibility to be used for inverse profiling.

2. METHOD OF ANALYTICAL PROFILING OF THE RACK TYPE CUTTER

The parameters of describing the parts including the complex helical surfaces. The parameters describing the helical surface are the profile frontal and the helix. The profile is described by the polar coordinates r, δ and ξ the angle between the polar radius and the tangent to the profile in the point (figure 1) [8].

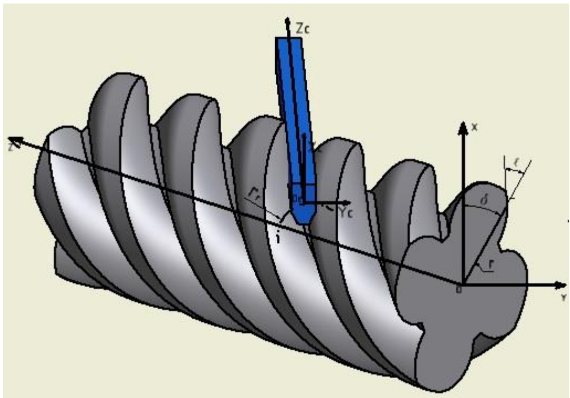

Fig. 1. Gearing of the rotor and the rack with their coordinates systems

Defining the basic rack-type cutter. The surface of the basic rack-type cutter is described by the rectilinear of its profile.
We attach a system of coordinate's $x_cy_cz_c$ in the following way:
-The origin of the system is placed in the initial plan from the intersection point 0_c of the line $0_c0_c'$ which is tangent to the initial cylinder of the part and the axis x_c of the system of coordinates $x_cy_cz_c$, joint to the part; the axis x_c0_c is perpendicular on the initial plan and it is oriented from the axis of the part to the exterior;

-The axis z_c is oriented along the tooth of the basic rack-type cutter. The initial position of the profile is considered in the frontal plan $x_c0_cy_c$. This plan is perpendicular to the direction of the teeth of the tool and for this reason it is called a normal plan; the corresponding profile is called the normal profile of the tool. In order to solve the problems generating the part in each point of calculus of the tool the following parameters must be known: the coordinates x_c si y_c; the angle α_C between the axis x_c and the tangent to the profile and the curvature radius of the profile ρ_C. The sign of the angle α_C coincides with the one of the derivate dy_c/dx_c, according to the relation tg $\alpha_C = dy_c/dx_c$, and the sign of ρ_C coincides with the sign of the second derivate $d2y_c/dx_c^2$.

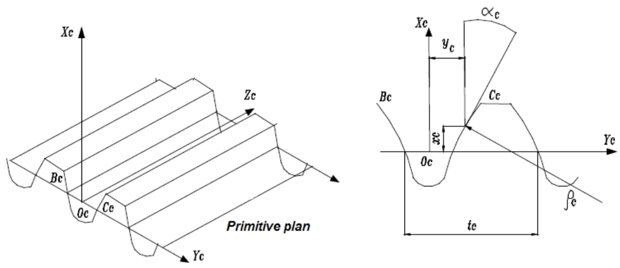

Fig. 2. The pitch t_c of the teeth of the tool in the normal plan $x_c0_cy_c$ is called normal pitch

3. THE NUMERICAL DETERMINATION OF THE PROFILE OF THE BASIC RACK-TYPE CUTTER

The numerical determination of the profile of the basic rack-type cutter can be done by using the geometrical method and analyzing the spatial gearing of the helical surface of the part with the surface of the basic rack-type cutter. Such a view on the laws of gearing of the part and of the basic rack-type cutter allowed the reduction of the generating problems of the teeth parts. The figure 3 presents the helical surface if the tooth part in gearing with the basic rack-type cutter.

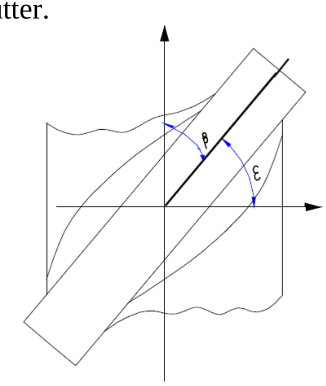

Fig. 3. Angle of the rack

The rake angle of the helix was noted with ε and its value can be determined from the following relations:

$$\begin{cases} r_r = ptg\beta \\ r_r = pctg\varepsilon \\ \varepsilon = \dfrac{\pi}{2} - \beta \end{cases} \quad (1)$$

where: r_r = 6mm, the initial radius of the part; β- the angle between the axis of the part and the axis O_cz_c of the rake.

The formula of passing from the system of coordinates xyz in the system of coordinates $x_cy_cz_c$ is determined by considering the basic deviations:

-the rotation with the angle ε around the axis 0x;

-the translation along the axis 0_x:

$$\begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} A^4(\varepsilon)A^1(r_r) - \begin{pmatrix} x_c = x - r_r \\ y_c = y\sin\varepsilon - z\cos\varepsilon \\ z_c = y\sin\varepsilon + z\sin\varepsilon \\ 1 \end{pmatrix} \quad (2)$$

Replacing in these formulae the values of the coordinates x, y, and z from the equation:

$$\begin{cases} x = r\cos(\delta + \varphi) \\ y = r\sin(\delta + \varphi) \\ z = p\varphi \end{cases} \quad (3)$$

we obtain the equation of the helical surface of the tooth of the part redressed in the system of coordinates of the rack.

$$\begin{cases} x_c = r\sin\mu - r_r \\ y_c = r\sin\mu \cdot \sin\varepsilon - p\varphi\cos\varepsilon \\ z_c = r\sin\mu\cos\varepsilon + p\varphi\sin\varepsilon \end{cases} \quad (4)$$

where, $\mu = \delta + \varphi$ (5)

Using the notations as:

$$\begin{cases} \tau = \mu + \xi \\ v = r\sin\xi \\ u = r\cos\xi \end{cases} \quad (6)$$

the equation (4) becomes:

$$\begin{cases} x_c = u\cos\tau + v\sin\tau - r_r \\ y_c = (u\sin\tau - v\cos\tau)\sin\varepsilon - p\varphi\cos\varepsilon \\ z_c = (u\sin\tau - v\cos\tau)\cos\varepsilon - p\varphi\sin\varepsilon \end{cases} \quad (7)$$

By cutting with a plane perpendicular to the y-axis (z) curve is obtained the tangency between spatial curve corresponding to the part, and straight line corresponding to the rack. Expressing mathematically this condition we obtain the following relations:

$$\begin{cases} x_c = r\cos\mu - r_r \\ y_c = (r\sin\mu - r_n\varphi)\sin\varepsilon \end{cases} \quad (8)$$

for profile of the rack.

4. DESCRIBING THE CALCULUS PROGRAM FOR THE DETERMINATION OF THE PROFILES OF THE CONDUCTING ROTOR AND OF THE CONDUCTED ROTOR

In term of the calculus methodology used for the determination of the profiles of the conducting and of the conducted rotor, a set of two programs was created to estimate the points of the profiles of the conducted/conducting rotor.

We started from the profile of the conducting rotor which contained a number of 4 curves: E_1D_1- circular arc, F_1E_1 – circular arc, N_1F_1- arc epicycloid, M_1N_1- circular arc (figure 4).

The calculus relations used in the creation of the program were the following ones:

For the curve M_1N_1- circular arc with the radius r and the centre in O, on the circle of radius r_1.

$$\begin{cases} x_1 = r\cdot\cos\theta + r_1 \\ y_1 = r\cdot\sin\theta \end{cases} \quad (9)$$

For the curve N_1F_1- arc of epicyclic resulted from the rolling of the point F_2 on the circle with the radius r_1.

$$\begin{cases} x_1 = A\cdot\cos(\phi-\psi_1) - r_2\cdot\cos\left[\left(1+\frac{1}{1}\right)\phi-\psi_1\right] \\ y_1 = -A\cdot\sin(\phi-\psi_1) - r_2\cdot\sin\left[\left(1+\frac{1}{1}\right)\phi-\psi_1\right] \end{cases}$$

(10)

For the curve E_1F_1 – circular arc with the radius r_0 and the center on the circle with the radius r_1.

$$\begin{cases} x_1 = r_1\cdot\cos\psi_3 - r_0\cdot\cos(\alpha-\psi_3) \\ y_1 = r_1\cdot\cos\psi_3 - r_0\cdot\cos(\alpha-\psi_3) \end{cases} \quad (11)$$

For the curve E_1D_1- with the radius R_{i1} and the center in O_1 we have:

$$\begin{cases} x_1 = R_{i1}\cdot\cos\phi \\ y_1 = R_{i1}\cdot\cos\phi \end{cases} \quad (12)$$

The section $M_1B_1C_1D_1$ from the curve is symmetrical after the axis Ox, so the curves will have the same analytical expression.

Considering that the angle δ is negative, the description parameters of the profile were analogously determined for the other extremity of the tooth.

The profile of the conducted rotor contains the following curves (figure 5):

A_2B_2- circular arc with the radius r_1 and the center in the point O;

$$\begin{cases} x_2 = r\cdot\cos\theta + r_2 \\ y_2 = r\cdot\sin\theta \end{cases} \quad (13)$$

B_2C_2- circular arc with a radius r_0 and the center with a radius r_2;

$$\begin{cases} x_2 = r_2\cdot\cos t_4 - r_0\cdot\cos(\alpha-t_4) \\ y_2 = r_2\cdot\sin t_4 - r_0\cdot\sin(\alpha-t_4) \end{cases} \quad (14)$$

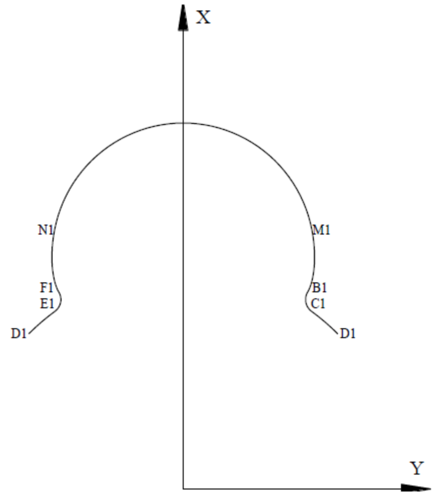

Fig. 4. Profile of the conducting rotor

C_2D_2- circular arc with the radius R_{e2} and the center O_2;

$$\begin{cases} x_2 = -R_{e2}\cdot\cos\varphi \\ y_2 = -R_{e2}\cdot\sin\varphi \end{cases} \quad (15)$$

The section $A_2F_2E_2D_2$ is symmetrical to the axis OX, the determination method of the describing parameters of the frontal profile of the helical surface is similar to the one used in the case of the conducting rotor.

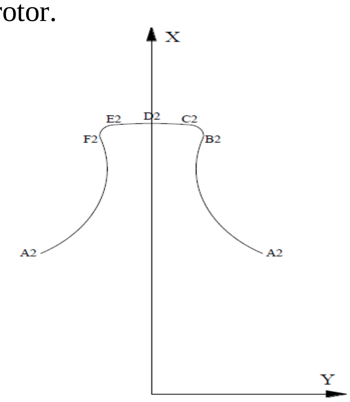

Fig. 5. Profile of the conducted rotor

5. THE MODEL OF GRAPHIC ANALYTICAL METHODS TO DETERMINE THE TOOL PROFILES FOR THE MANUFACTURING OF THE COMPLEX HELICAL SURFACES

The use of the graphic analytical methods implies major difficulties due to the large number of points necessary to describe the profile of the tools. In order to support the work of the tool designing engineer in the generating process and to use the facilities offered by the AutoCAD environment, specific graphic methods were conceived both for the profile of the tools and for the generation of the helical surfaces.
The methods taken into consideration are based on the principles of the methods used in the chip removal process, known as: mobile straight line and fix straight line.
The first method was followed by the creation in the AutoCAD environment of a package of programs to determine graphically the profile of the rack interlinked with the profile of a complex helical surface, [9]. This method includes the following steps:
-protracting the profile in AutoCAD by determining its geometrical parameters through the analytical calculus of the points of discretization of the curves that generate it;

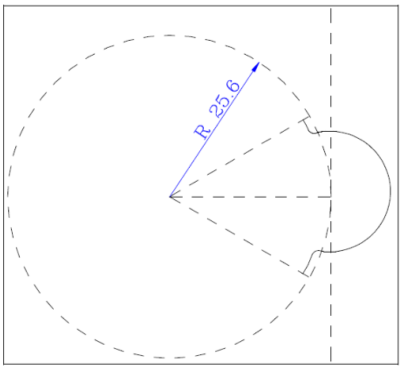

Fig. 6. The initial position of the rotor and rack

-representing a blank rack for which the reference line is tangent to the rolling circle;

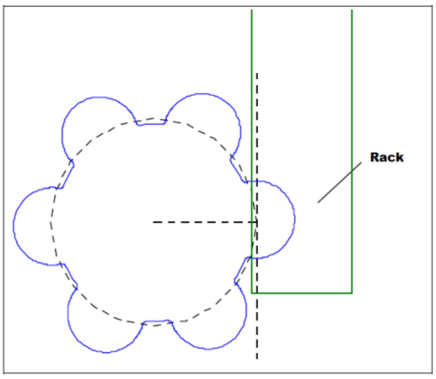

Fig. 7. Start of rack movement

-introducing the parameter describing the rolling process. The rotation angle of the profile is used in this case;
-correlating the movements necessary for the rolling process, the rotation movement of the profile and the translation movement of the rack and the generation of the rolling. The correlation is realized through a program in AutoLISP (two random positions of this process were represented in figure 8 and figure 9).

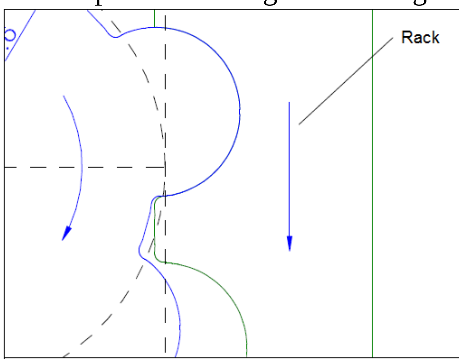

Fig. 8. The intermediate position 1 during rolling movements

-removing the distance part from the rack for the parts that overlap between the two elements to the point where the rotor tooth jumps out of the rack rolling;

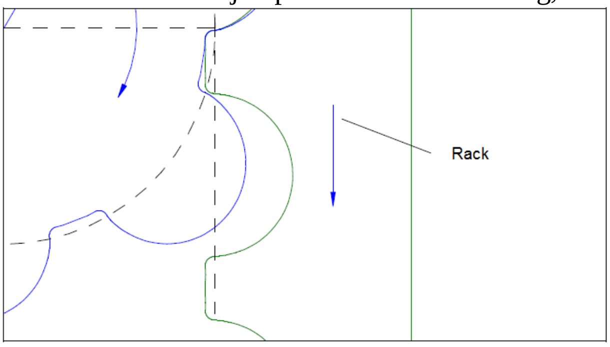

Fig. 9. The intermediate position 2 during rolling movements

-comparing the profile of the graphically generated rack to the analytically determined one (figure 10). A strong correspondence can be detected between the two profiles.

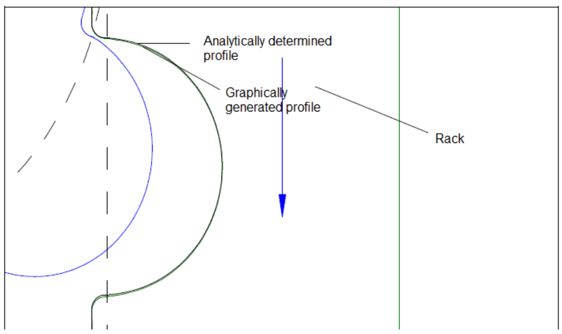

Fig. 10. Comparison between analytical and graphical profile

Some of the disadvantages of the method presented above consist of the fact that due the movement executed by the rack, its shifting and the final position will depend on the profile of the complex helical surface; this will obviously make difficult the process of comparing the analytical and graphic profile of the tool, comparison based on the overlapping of the two profiles in relation with the same system of coordinates. In order to remove this obstacle, another method was designed to allow the generation of the rack profile without modifying the position of the rack. The program is using the principle of rotation with a fix straight line, when the profile of the rotor executes both the rotation and the translation movement. This method involves the steps below:

-Drawing the frontal profile of the helical surface in the system of coordinates;

-Projecting a rectangle representing the blank of the rack;

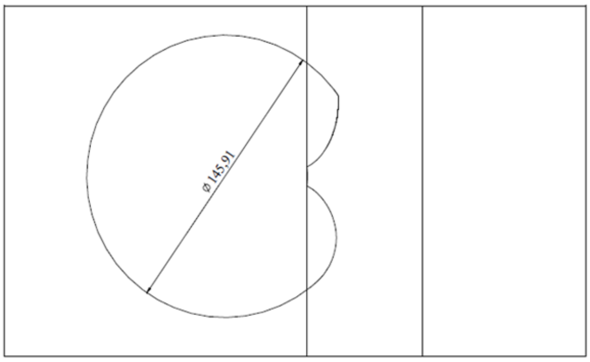

Fig. 11. The start position of rolling movement

-Introducing the parameters for the rotation process;

-Generating the interdependent movements through an AutoLISP program needed for the rotation process and for the removal from the "blank" of the rack of the surfaces the profile of the helical surface is overlapping with during the rotation process;

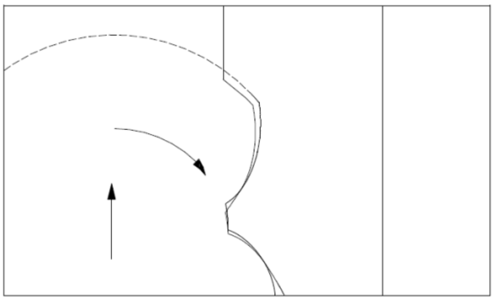

Fig. 12. The intermediate position of rolling movement

-Comparing the profiles and determining the maximal deviation.

The analysis of the results underlined a high precision; the graphical method provides solutions for the completion of the tool profile even when the use of the analytical method leads to tools with discontinuities.

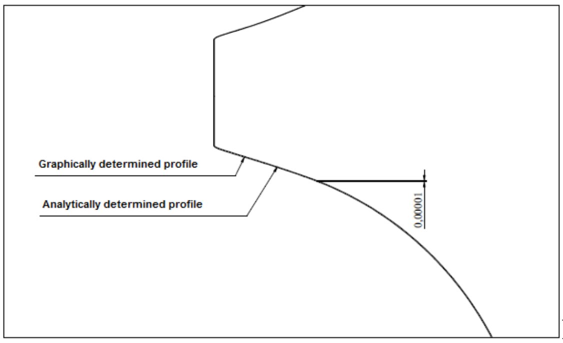

Fig. 13. Comparing the profiles and determining the maximal deviation

6. ANALYTICAL DETERMINATION OF THE PROFILES NEEDED FOR THE CONSTRUCTION OF THE RACK CUTTER TOOL FOR THE HELICAL COMPRESSOR WITH SYMMETRIC CIRCULAR PROFILE

Based on the mathematical model of the numerical determination, we focused on the delimitation of the profiles needed for the construction of the hob for the manufacturing of the helical surfaces of a helical compressor with symmetric circular profile. Following the previously presented methodology, the steps below were considered:

-Determining the parameters of description of the frontal profile of the complex helical surfaces of the rotors;

-Settling the parameters of installation of the rack;

-Determining numerically the profile of the rack;

In the case of the two frontal profiles, the parameters of the profile r, δ, β, ξ and the Cartesian coordinates x and y are presented in the table 1 and in the table 2. A limited number of points were considered on each curve due to the limited space of the present paper. 150 points were used in the program of calculus for the selected rotors, on the profile of the conducting rotor and 128 points were used on the profile of the conducted rotor.

The profiles of the racks for these profiles were determined using the methodology and the relationships (7).

A particular study of the points from the profile was developed in order to determine the profile of the rack; the priority points were identified as well as the ones involving difficulties during the manufacturing process. In the case of the two rotors, these are: B_1 for the conducting rotor and B_2 for the conducted rotor. Using the given relationships, the rolling radii of the two rotors were determined $r_{r1} = 25.6$ and $r_{r2} = 38.4$. The profiles of the racks are presented in tables 3 and 4. The graphic representation of these racks was realized in figure 13 and figure 14.

Table 1. Profile frontal of conducting rotor

Pt. No	Curve	r_r	δ	β	ξ	x	y
0	E1-D1	24.0000	0.7854	-0.7854	1.5708	16.9706	16.9706
4	E1-D1	24.0000	0.6284	-0.9424	1.5708	19.4158	14.1076
5	F1-E1	24.0000	0.6284	-0.9424	1.5708	19.4158	14.1076
15	F1-E1	24.4764	0.5829	-0.1727	0.7555	20.4353	13.4719
26	F1-E1	25.6000	0.5659	0.5971	-0.0312	21.6098	13.7250
27	N1-F1	25.6000	0.5659	-1.0050	1.5708	21.6098	13.7250
38	N1-F1	27.5821	0.5455	0.1509	0.3946	23.5791	14.3109
47	N1-F1	31.6380	0.4635	-0.1886	0.6521	28.3000	14.1446
48	M1-N1	31.6380	0.4635	-0.1886	0.6521	28.3000	14.1446
73	M1-N1	40.0000	0.0000	1.5708	-1.5708	40.0000	0.0000
98	M1-N1	31.6380	-0.4635	0.1886	-0.6521	28.3000	-14.1446
99	M1-B1	31.6380	-0.4635	0.1886	-0.6521	28.3000	-14.1446
110	M1-B1	26.9446	-0.5543	-0.2263	-0.3280	22.9091	-14.1839
120	M1-B1	25.6000	-0.5658	1.0050	-1.5708	21.6098	-13.7250
121	B1-C1	25.6000	-0.5658	-0.5971	0.0313	21.6098	-13.7250
130	B1-C1	24.5694	-0.5795	0.0957	-0.6752	20.5574	-13.4555
142	B1-C1	24.0000	-0.6283	0.9424	-1.5708	19.4158	-14.1076
143	C1-D1	24.0000	-0.6283	0.9424	-1.5708	19.4158	-14.1076
150	C1-D1	24.0000	-0.7854	0.7854	-1.5708	16.9706	-16.9706

Table 2. Profile frontal of conducted rotor

Pt. No	Curve	r_r	δ	β	ξ	x	Y
1	A2-F2	24.00000	0.52360	-1.04720	1.57080	20.78461	12.00000
11	A2-F2	27.21691	0.24205	-0.49433	0.73638	26.42347	6.52384
26	A2-F2	38.40000	0.14637	0.33498	-0.18862	37.98941	5.60041
27	F2-E2	38.40000	0.14636	0.12553	0.02083	37.98943	5.60027
38	F2-E2	39.42508	0.13627	-0.57479	0.71106	39.05960	5.35575
53	F2-E2	40.00000	0.10469	-1.46610	1.57080	39.78099	4.18006
54	E2-D2	40.00000	0.10469	-1.46610	1.57080	39.78099	4.18006
59	E2-D2	40.00000	0.05235	-1.51845	1.57080	39.94521	2.09289
64	E2-D2	40.00000	0.00000	1.57080	-1.57080	40.00000	0.00000
65	D2-C2	40.00000	0.00000	-1.57080	-1.57080	40.00000	0.00000
70	D2-C2	40.00000	-0.05230	1.51845	-1.57080	39.94521	-2.09289
75	D2-C2	40.00000	-0.10460	1.46610	-1.57080	39.78099	-4.18006
76	C2-B2	40.00000	-0.10460	1.46610	-1.57080	39.78099	-4.18006
89	C2-B2	39.50015	-0.13450	0.63845	-0.77298	39.14328	-5.29768
102	C2-B2	38.40000	-0.14630	-0.1255	-0.02083	37.98940	-5.60027
103	B2-A2	38.40000	-0.14630	-0.3349	0.18862	37.98941	-5.60041
116	B2-A2	28.47093	-0.20680	0.38375	-0.59060	27.86399	-5.84736
128	B2-A2	24.00000	-0.52360	1.04720	-1.57080	20.78461	-12.00000

Table 3. Rack profile points of conducting rotor

Pt. no.	x_c	y_c
0	-1.6000	20.1062
4	-1.6000	16.0860
5	-1.6000	16.0860
15	-1.1489	14.9725
26	-0.0500	14.4868
27	0.0000	14.4858
38	0.8236	14.4287
47	2.6999	14.1447
48	2.6999	14.1447
73	14.4000	0.0000
98	2.6999	-14.1447
99	2.6999	-14.1447
110	0.5519	-14.4544
120	0.0000	-14.4858
121	-0.0500	-14.4868
130	-1.0599	-14.8875
142	-1.6000	-16.0860
143	-1.6000	-16.0860
150	-1.6000	-20.1062

Table 4. Rack profile points of conducted rotor

Pt. no.	x_c	y_c
1	-14.4000	20.1062
11	-12.2547	12.5441
26	-2.7001	5.9618
27	0.0000	5.6202
38	1.0054	5.2649
53	1.6000	4.0201
54	1.6000	4.0201
59	1.6000	2.0102
64	1.6000	0.0000
65	1.6000	0.0000
70	1.6000	-2.0083
75	1.6000	-4.0166
76	1.6000	-4.0166
89	1.0826	-5.1982
102	0.0000	-5.6202
103	-2.7001	-5.9618
116	-11.3454	-11.2381
128	-14.4000	-20.1062

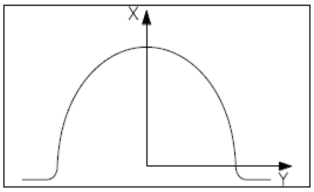

Fig. 13. Analytical profile of rack for conducting rotor

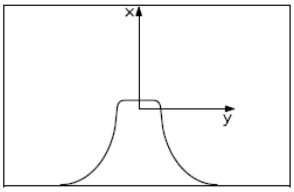

Fig. 14. Analytical profile of rack for conducted rotor

7. CONCLUSIONS

The new methods to obtain the profile of rack type cutter are asked by the time for design of the tools (special for the hob) used for manufacturing helical surfaces. They can be combined with analytical methods and used for simulation of the processes and for design also the rotors also the rolling tools. The accuracy of the profiling is at very high level and can replace analytical methods with the development of software for design will create new possibility of optimization of the rotors of helical surfaces and the tools for the rotors.

8. REFERENCES

1. Abrudan G., (1971), *Generalizarea metodei de determinare a erorilor de profil rezultate la prelucrarea pieselor cu scule profilate*, Ph.D. Thesis, Politehnic Institute of Cluj Napoca.
2. Ilea H. (1991), *Studiul comparativ al compresoarelor frigorifice elicoidale cilindrice şi globoidale şi realizarea practică a prototipului cu parametri funcţionali îmbunătăţiţi*, Ph.D. Thesis Politehnic Institute of Cluj, pp. 26.
3. Kirsanov G.N., Lazebnik I.S. (1983), *Profilirovanie vîsokoproizvoditelnîh cerviacinîh tverdosplavnîh frez.În: Stankii instrument*, No. 10, pp. 22-23.
4. Marxmeyer R. (1985), *Fräser zum Bearbeiten von Schraubenverdichter-Rotoren Werkstatt und Betrieb* 118, 5, pp. 259-260.
5. Oancea N., Neagu M., Arbureanu C. (1992), *Metode numerice pentru profilarea sculelor*, Volume II, Dunarea de Jos University of Galati, pp. 160.
6. Plahteanu B., Pleşu Gh., Afrăsinei M., (1987), *Aspecte ale determinării parametrilor de instalare optimi la prelucrarea suprafeţelor elicoidale asimetrice cu scule disc*, Volume "AGMUS" 87, pp. 154-161.
7. Plahteanu B., Pleşu Gh., Grigoriu M., Luca L., (1987), *Aspecte ale proiectării frezelor melc pentru prelucrarea rotorilor compresoarelor elicoidale*, Volume "Scule-87", The Second National Tools Symposium, Bucharest.

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