A NUMERICAL INVESTIGATION INTO THE CAPSIZE PHENOMENON OF A VESSEL

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Abstract: Each year, almost one hundred vessels are lost due to the capsizing, particularly in high seas, leading to heavy losses of human lives and ships. This is why the stability against capsizing is a fundamental requirement when designing a ship. In our work, the second-order nonlinear differential equation developed by Thompson et al. is taken as a model for a vessel capsize in periodic beam seas. It describes the behavior of a ship that is simultaneously heaving, swaying and rolling in waves. With the forcing amplitude as bifurcation parameter, the analyzed system exhibits either periodic or chaotic behavior, the route to chaos being realized by a period doubling sequence of periodic motions. Some accepted indicators, like bifurcation diagrams, phase planes and Poincare maps have been computed and they confirm the transition from order to chaos. The paper investigates also the fractal erosion of safe basin of attraction and proposes a geometric way to evaluate quickly this process leading to capsize.
Key words: Capsize Direct and Parametric Excitation, Order and Chaos, Safe basin.

1. INTRODUCTION

By capsizing or keeling over of a ship we understand that situation when the ship is turned on its side or it is upside down. In the language of nonlinear dynamics, capsizing is a transition from a stable equilibrium point near the upright position to a stable equilibrium point near the upside-down position. Despite of an extensive research on the capsizing problem of a vessel, not much practical progress has been made so far and a large number of ships continue to sink because of this phenomenon. This is due of the huge number of factors involved, including severe winds and waves, water on deck, liquid cargo, lack of information about environmental conditions during the accident or resonance conditions, [1].

According to the IMO (International Maritime Organization) there is a necessity for proposing mathematical models that could simulate the ship's capsizing taking into account as much as possible of these factors. Such examples can be found in Kan and Taguchi [2], Soliman and Thompson [3], Umeda and Hamamoto [4], and others. Most of these approaches consider only the ship's rolling motion, so the mechanical system can be reduced to a forced nonlinear oscillator with linear/ nonlinear damping and restoring force.

In the present study, the following equation, derived by Thompson et al., [5], is taken as a model for a vessel capsize in periodic beam seas:

$$\ddot{x}+\beta\dot{x}+\left(x-x^2\right)\left(1+G\cos\omega t\right)=F\sin\omega t \qquad (1)$$

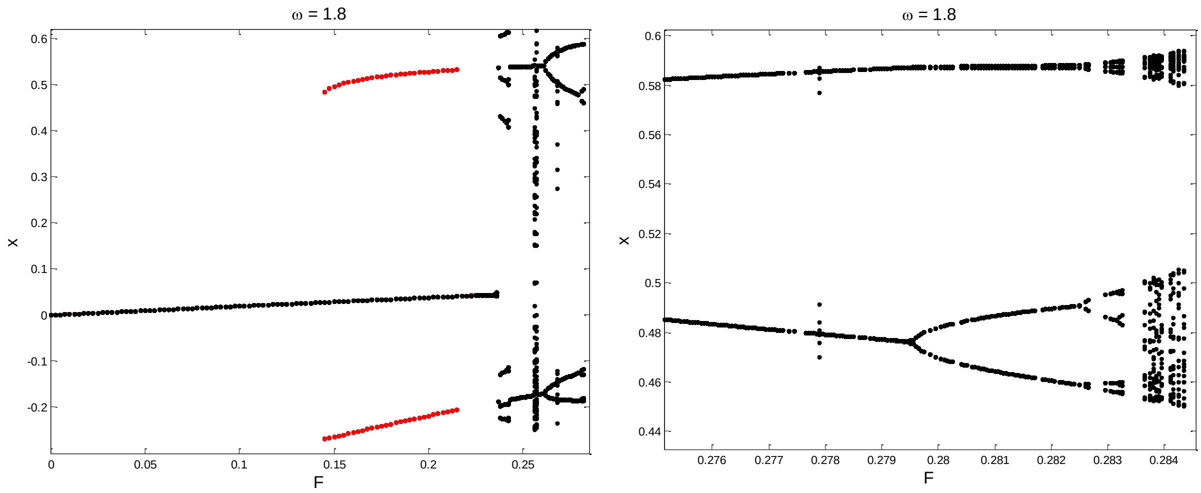
Where:

$$x=\frac{\theta}{\theta_V},\ \omega=\frac{\omega_f}{\omega_n},\ F=-\frac{Ak\,\omega^2}{\theta_V},\ G=-Ak \qquad (2)$$

Here, represents the roll angle, θ, the angle of vanishing stability, θ_V, the non-dimensional damping coefficient, the wave frequency, ω, the natural frequency of the boat, ω_n, A the wave height, and k the wave number. A dot denotes differentiation with respect to time. The details about Eq. 1 can be found in [5]. Despite of its simplicity, Eq. 1 shows a wide spectrum of qualitatively distinct types of behaviors, including steady-state solutions, jumps to resonance, or period doubling cascades leading to chaos. This rich dynamics will be investigated in the next sections.

2. BIFURCATION DIAGRAMS, PHASE PLANES AND POINCARE SECTIONS

The response of the system (1) has been investigated numerically by use of a Runge – Kutta - Gill procedure with constant step for the following parameters: $\beta=0.1, \omega=1.8, G=5F$ [5]. With the forcing amplitude F as bifurcation parameter, it was found that the analyzed system exhibits either periodic or chaotic behavior, the route to chaos being realized by a doubling sequence of periodic motions. The bifurcation diagram $F-x$ showing the different bifurcation points is given in Fig. 1. To construct this diagram, we started from equilibrium conditions ($x(0)=\dot{x}(0)=0$) and plotted x at every one forcing cycle with the same phase angle. The first 200 cycles were discarded to avoid transients.



(a) (b)

Fig. 1. a) Bifurcation diagram *F* - *x* for $\beta = 0.1, \omega = 1.8, G = 5F$;
b) The upper – right part of (a) is zoomed for details

For small forcing amplitude *F* the system executes periodic oscillations with period corresponding to the forcing period $T = 2\pi / \omega$, as shown in Fig. 2(a). As *F* is gradually increased different types of periodic motions are obtained. Starting with *F* = 0.235, the period 1 motion bifurcates into a period 2 motion. The phase – plane $x - \dot{x}$ indicates a continuously and significantly change of the trajectory's shape (see Figs. 2 (b)-(d)). For further increase in forcing amplitude period 4, 8 and 16 are obtained, as illustrated in Figs. 2 (e)-(g). When the forcing amplitude outruns the value 0.286 the response becomes chaotic, as given in Fig. 2(h). In all the panels of Fig. 2, the points of the Poincare map corresponding to one period of forcing are also plotted in the phase planes, and are indicated by a blue big dot. Finally, if F exceeds 0.287 the scheme becomes unstable, and the amplitude goes to infinity (the ship is in capsizing state).

The behavior described above remains unchanged for other initial conditions around equilibrium position. The point $(x(0), \dot{x}(0))$ is considered safe if the amplitude of the displacement $x(t)$ not exceeds unity, that means the roll angle θ remains smaller than the angle of vanishing stability, θ_V. For a non-safe point, the associated trajectory goes out in the phase plane, like a spiral with expanding amplitude [6]. For a given configuration of parameters (β, F, G, ω), the set of all initial conditions that do not lead to capsize is called safe basin of attraction. We will comment more on this topic in the next section.

3. SAFE BASIN OF ATTRACTION AND INTEGRITY CURVES

Safe basins can be generated numerically by various techniques including cell-to-cell mapping and coarse grid-of-start method. The simulation show that an increasing of the forcing amplitude leads to a process of fractal erosion of the safe basin, finished with a sharp decrease of safe area.

To illustrate this, Eq. 1 was solved numerically just up to ten cycles of the forcing. Our investigation was restricted at this time interval because experiments and numerical simulations have showed that if escape has not occurred within 8 – 10 cycles than it is unlikely to appear in the following cycles.

The initial conditions $(x(0), \dot{x}(0))$ have been selected from a vast set having 40,401= 201 x 201 elements, obtained by dividing the rectangle [- 0.8, 1.2] x [- 0.9, 0.9] in equally spaced segments. Each point is tested against the escape criterion, and classified as safe or not. If it is safe then a small black rectangle is drawn around it in the phase plane $(x, \dot{x})$, otherwise the rectangle is maintain white. In this way, the safe basin is given by the black area in the phase plane.

The fractal erosion experienced by the safe basin could follow different routes [7]. Fig. 3 presents a slow and permanent reduction of safe basin owing to the increasing of the forcing amplitude *F*.

Every panel in Fig. 3 requires about 80 min CPU time on our computer, so the computational cost for having a complete image of the fractal erosion of safe basin for a given (β, ω, G) pair is remarkably high. To avoid this, we plot instead the intersection between safe basins and *x*, respectively $\dot{x}$ axis, for any value of excitation's amplitude *F* that leads to emptiness safe basins.

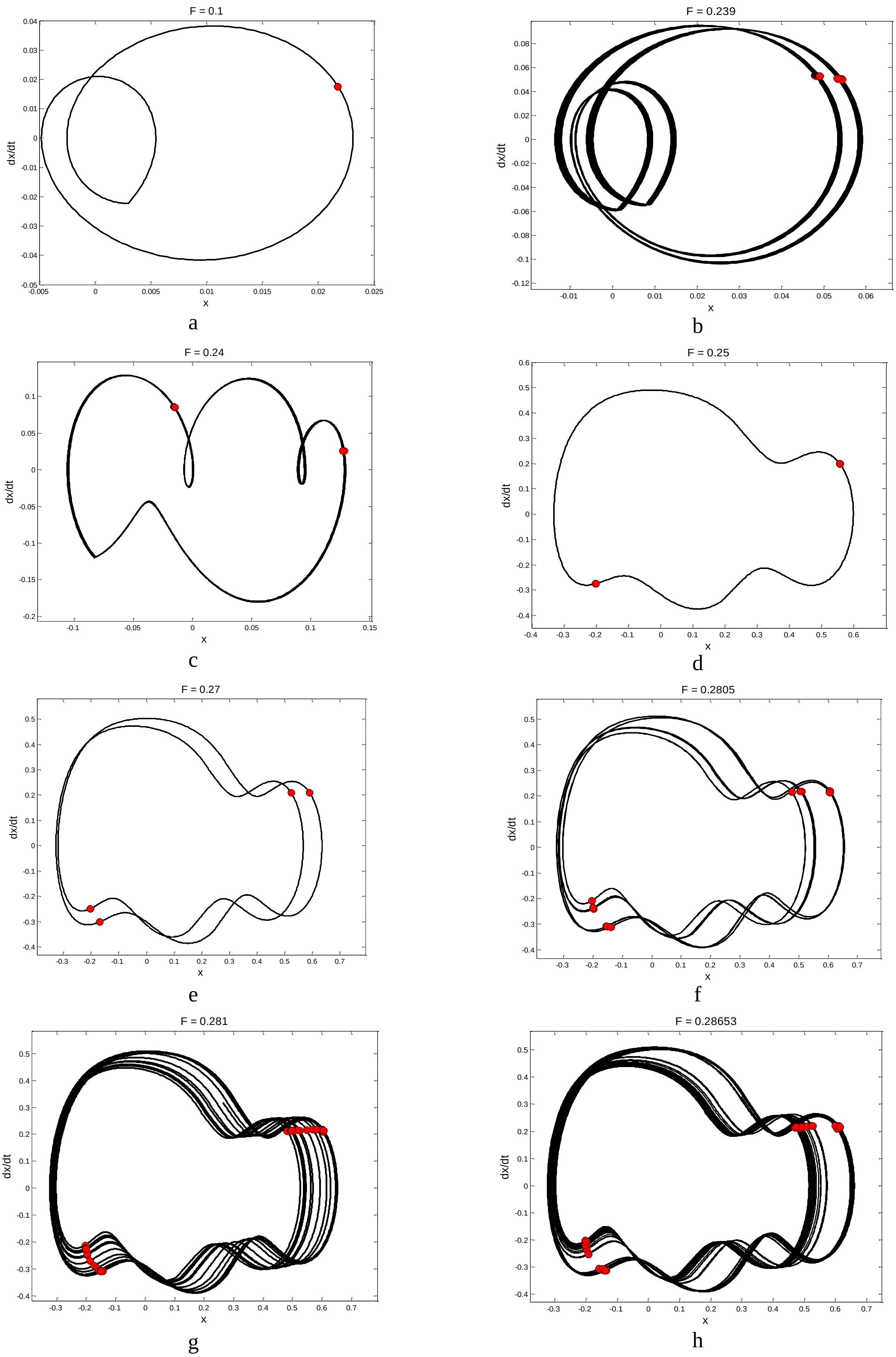
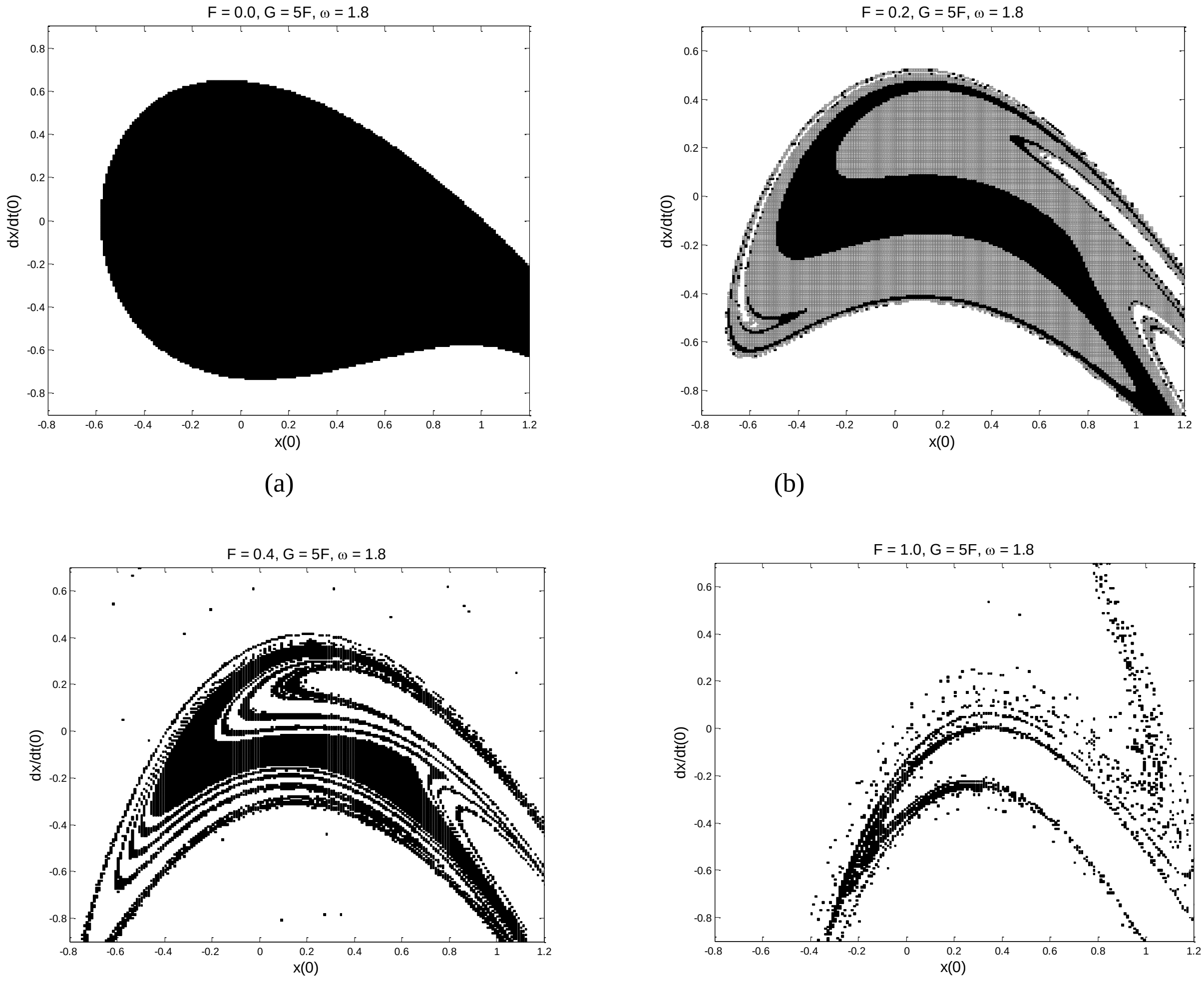


Fig. 2. Phase planes and Poincare maps for $\beta = 0.1, \omega = 1.8, G = 5F$.

(a) Period 1 orbit (F = 0.1); (b) Period 2 orbit (F = 0.239); (c) Period 2 orbit (F = 0.24); (d) Period 2 orbit (F = 0.25); (e) Period 4 orbit (F = 0.27); (f) Period 8 orbit (F = 0.2805); (g) Period 16 orbit (F = 281); (h) Chaotic orbit (F = 0.2853)

The results are given in Fig. 4. Panels have been constructed for 200 values of *F* chosen uniformly in the mentioned interval. Although less than an hour have been necessary for the whole computational process (for every panel), we have now a clear image of both exterior and interior erosion of safe basin.



(c) (d)

Fig. 3. Erosion of the safe basin for Eq.(1) with $\beta = 0.1, \omega = 1.8, G = 5F$:
(a) *F* = 0.0; (b) *F* = 0.2; (c) *F* = 0.4; (d) *F* = 1.0

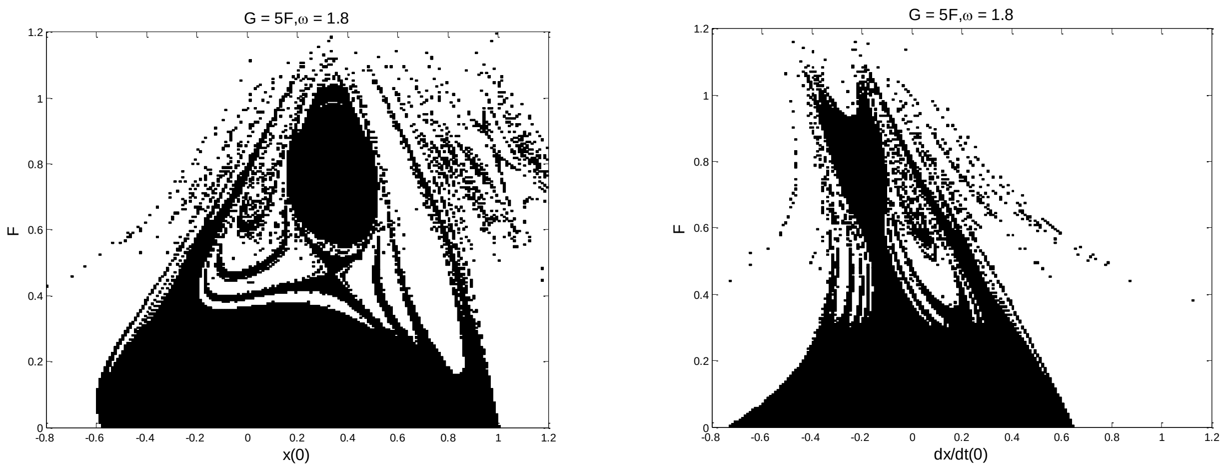


Fig. 4. Intersection between safe basins and: a) *x* axis; b) $\dot{x}$ axis, for $\beta = 0.1, \omega = 1.8, G = 5F$

The integrity curves show the relative influence of wave excitation's amplitude F on capsize relative to vessel safety in the absence of incident waves. They could be generated by plotting the safe area, normalized to unity at F = 0, over the same ranges of F values used in Fig. 4. The simulations results are displayed in Fig. 5 (the asterisks).

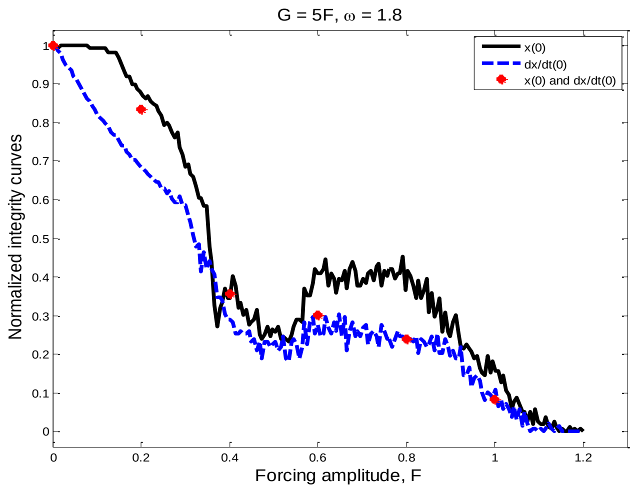


Fig. 5. Normalized integrity curves for $\beta = 0.1, \omega = 1.8, G = 5F$

The above-mentioned figure contains also the curves giving the dependence of safe segments (normalized to unity at F = 0) along x and $\dot{x}$ axis, respectively, on excitation's amplitude. Although the computational cost for constructing these curves is small compared with that needed for a complete integrity curve, these additional curves provide practically the same behavior for the safe basin. Looking for a compromise between computational cost and accuracy of the results, the use of additional curves seems to be very promising.

4. CONCLUSIONS

In this study, a thorough numerical analysis of the capsize equation derived by Thompson et al. is performed. It includes two possible explanations of capsize phenomenon, namely the fractal erosion of the safe basin of attraction and the change of the vessel's motion from a periodic to a chaotic one, throw means of a period doubling sequence of periodic motions. Our study has shown that the fractal erosion of the safe basin investigated in transient conditions, starts considerably before the final loss of ship's stability and could be seen as a first signal of the complex chain of events leading to capsizing. It was also proven that the period bifurcation phenomenon is a precursor of the chaotic behavior and, finally, of ship capsizing. The first bifurcation point, from a period T-orbit to a period 2T-orbit, is accompanied by a significant jump of the roll amplitudes. The roll angle becomes dangerously close to the angle of vanishing stability of the vessel, which is unacceptable from an operational perspective. Additionally, the paper proposes a new geometric way to estimate the process of fractal erosion of safe basin, by considering the intersection of safe basin with the axis defining the roll angle and roll velocity.

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