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ON INFLUENCE OF MODEL PARAMETERS ON TIME-DEPENDENT QUEUE-SIZE DISTRIBUTION IN A PRODUCTION MODEL WITH FAILURES AND BATCHED ARRIVALS

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Abstract: Sensitivity of the fleeting behaviour of the distribution of number of jobs waiting in the line in a single-machine manufacturing line with machine damages, on changing of the "input" system parameters is investigated numerically. As a mathematical model of the production system a finite-buffer queueing system with batched Poisson arrivals, exponential processing times, and with cyclic succession of failure-free and repair periods is assumed. We study the influence of such parameters as the intensity of job arrivals, sizes of the arriving batches of jobs, processing time and failure-free and repair period durations. In particular, we estimate the relaxation time of the fleeting distribution of the number of jobs waiting in the line in dependence on the initial buffer state for the system with double arrivals (the arriving batches consist of two jobs) and being critically loaded. Next, we investigate the transitory progress of this characteristic in a function of intensity of job arrivals, the processing (service) speed and different values of mean failure-free and repair times. Finally, we investigate dependence of the stationary distribution of the number of jobs on the function describing the size of the arriving portion of jobs. Numerical analysis of different scenarios of the system behaviour is performed by using the *Mathematica 9.0* environment.
Key words: finite buffer capacity, manufacturing line, queue size, repair time, non-stationary state, working time.

1. PRELIMINARIES

The optimal utilization of the manufacturing line requires permanent monitoring of key reliability characteristics like e.g. failure-free and repair periods' durations, the level of the queue of jobs waiting for service or instantaneous queueing delay (see e.g. [1, 3-7, 9-10] for queueing models in manufacturing systems and systems with breakdowns). The analysis of the distribution of the number of jobs instantaneously present in the system, i.e. the distribution of the number of jobs not completely processed or waiting in the buffer queue, is particularly important from the point of view of optimization of system's operation. Time-dependent behaviour of the present jobs' distribution can be used in the performance evaluation of the manufacturing system. In particular, it can be helpful:
- in the projection of the buffer size to adjust it to service speed and intensity of arrivals;
- in the estimation of the level of manufacturing line's utilization;
- in the detection of untypical phenomena during the operation of the system, like periods of significantly increasing intensity of incoming jobs or, simultaneously, periods of low traffic level;
- in the prediction of buffer overflows during which the input flow of new jobs is timely suspended.

Obviously, to simulate the behaviour of the real manufacturing system in different scenarios and to protect it, eventually, against different-type adverse phenomena occurring in the real traffic, it is desired and necessary to investigate the sensitivity of the time-dependent queue-size distribution on "input'' parameters characterizing arrival flow of jobs, service speed, working period duration and repair time.

In the article we deal with the model of a single-line manufacturing process based on the finite-buffer queueing system with batched Poisson arrivals and exponentially distributed service times, in which successive uninterrupted periods of the machine operation are followed by repair times. We investigate the reaction of the total number of tasks present in the system on changes of the following system parameters: intensity of batched Poisson arrivals, probability distribution of sizes of job batches, parameter of exponential processing time, durations of failure-free and repair periods. Such an analysis can provide valuable information to make better the utilization of the manufacturing line, or adapt it to the level of the actual traffic of incoming jobs. Moreover, the observation of the time-dependent progress of successive probabilities is helpful in the estimation of the relaxation time, i.e. the time needed for the system to reach the equilibrium.

2. QUEUEING MODEL AND BASIC EQUATIONS

Let us consider a FIFO-type finite-buffer queueing model with the input flow of jobs described by a compound Poisson process with rate λ. Successive batches have sizes being independent random variables, where r_k is the probability that the batch consists of exactly k jobs. We assume jobs are being processed singly, with exponential processing time with mean $1/\mu$. The number of jobs simultaneously present in the system is bounded by a non-random value N, i.e. we have $N-1$ places in the buffer queue and one place "in a machine". If the size of the arriving batch is greater than the number of free places in the buffer, the buffer is being saturated and the remaining jobs are lost (we assume the partial batch acceptance strategy (PBAS)). We assume that the system may contain a number of jobs before the opening at time $t=0$ and that the machine is working at this time. After a failure-free time, which is exponentially distributed with mean $1/\gamma$, a generally distributed repair period with a distribution function $F(\cdot)$ follows and so on. A machine failure can occur only when the machine is busy with a processing of a job. Successive interarrival, processing, failure-free and repair times are assumed to be independent random variables. Introduce the following notation for the Laplace transform of the conditional transient queue-size distribution:

$$D_s(s,m) = \int_0^\infty e^{-st} P\{U(t) = m \mid U(0) = n\} dt \qquad (1)$$
$$\mathrm{Re}(s) > 0, t > 0, 0 \le m,n \le N$$

where $U(t)$ denotes the number of jobs present in the system at time t. In [8] the following system of equations was derived:

$$D_0(s,m) = \frac{\lambda}{\lambda+s}\left[\sum_{k=1}^{N-1} r_k D_k(s,m) + D_N(s,m)\sum_{k=N}^{\infty} r_k\right] + \frac{\delta_{m,0}}{\lambda+s} \qquad (2)$$

$$D_n(s,m) = h(s)\Bigg\{\lambda\left[\sum_{k=1}^{N-n-1} r_k D_{n+k}(s,m) + D_N(s,m)\sum_{k=N-n}^{\infty} r_k\right] + \mu D_{n-1}(s,m) + \gamma\Bigg[\sum_{k=0}^{N-n-1} g_k(s) \sum_{j=k}^{N-n-1} r_j^{k*} D_{n+j}(s,m) + D_N(s,m)\left(\sum_{k=N-n}^{\infty} g_k + \sum_{k=0}^{N-n-1} g_k(S) \sum_{j=N-n}^{\infty} r_j^{k*}\right) + I\{n \le m \le N-1\}\sum_{k=0}^{m-n} r_{m-n}^{k*} \nu_k(s) + \delta_{m,N}\left(\sum_{k=N-n}^{\infty} \nu_k(s) + \sum_{k=0}^{N-n-1} \nu_k(s) \sum_{j=N-n}^{\infty} r_j^{k*}\right)\Bigg] + \delta_{m,n}\Bigg\}, \quad 1 \le n \le N-1 \qquad (3)$$

and

$$D_N(s,m)[1 - h(s)(\lambda + \gamma f(s))] = h(s)[\mu D_{N-1}(s,m) + \delta_{m,N}(\gamma s^{-1}(1 - f(s)) + 1] \qquad (4)$$

where $\delta_{i,j}$ and $I\{A\}$ stand for the Kronecker delta function and the indicator of a random event A, respectively, and r_k^{i*} is i-fold convolution of the sequence (r_k) with itself. Besides, $f(\cdot)$ is the Laplace-Stieltjes transform of the distribution function $F(\cdot)$, and we use above the following notation:

$$g_k(s) = \int_0^\infty e^{-(\lambda+s)y} \frac{(\lambda y)^k}{k!} dF(y) \qquad (5)$$
$$h(s) = (\lambda + \mu + \gamma + s)^{-1}$$

$$\nu_k(s) = \int_0^\infty e^{-(\lambda+s)y} \frac{(\lambda y)^k}{k!}[1 - F(y)]dy \qquad (6)$$

In next sections we use the system Eq. 2 - Eq. 4 as the basic one for numerical computations.

3. ESTIMATION OF RELAXATION TIME IN DEPENDENCE ON INITIAL BUFFER STATE

Let us consider, as a first scenario, a model of a single-machine unreliable manufacturing line described by a finite-buffer queueing system, with the intensity of batched Poisson arrivals $\lambda=20$ (so, the mean interarrival time equals 0.05 time units) and the mean exponential processing time 0.025. Moreover, assume that $\gamma=1$ (so, the mean failure-free time equals 1) and that the repair time is exponentially distributed with mean 0.1. Each entering batch of jobs consists of exactly 2 items (we have $p_k=1$ for $k=2$). In consequence, the traffic load of the system equals (see e.g. [2]).

$$\rho = \frac{\text{mean processing time} \times \text{mean batch size}}{\text{mean interarrival time}} = \frac{0.025 \cdot 2}{0.05} = 1 \qquad (7)$$

so the system is critically loaded. We are interested in the estimation of the so called relaxation time, i.e. the time needed for the transient queue-size distribution to stabilize about the stationary value. Let us investigate particular case where *N=6* and the queue size *m=0*. By using the well-known Tauberian theorem (see e.g. [2]) we can find the stationary value π_0 of the proper transient probability, which is independent on the initial level of buffer saturation, from the following formula:

$$\pi_0 = \lim_{t\to\infty} P\{U(t)=0|U(0)=n\} = \lim_{s\to 0} s\cdot D_n(s,0) \quad (8)$$

Executing the appropriate computations in the *Mathematica 9.0* environment, we get $\pi_0=0.179865$. Solving the equations of the system Eq. 2-Eq. 4 and next inverting the Laplace transforms, we find the trajectories presented in Fig. 1 for three different levels of initial buffer saturation, namely *n=0, 3* and *6*. As one can observe, the queue-size distribution in each case stabilizes after about 0.8 time units. Evidently, before this time transient queue-size behaviour differs essentially for the considered three cases.

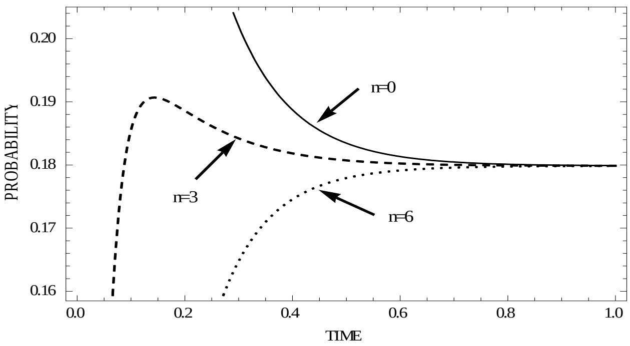

Fig. 1. Relaxation time of the transient queue-size distribution for *n=0,3 and 6*, for $\rho=1$

4. QUEUE-SIZE BEHAVIOUR IN A FUNCTION OF INTENSITY OF ARRIVALS AND SERVICE SPEED

Now, let us investigate the dependence of the transient queue-size distribution on the intensity of job arrivals and service speed. Let us, firstly, characterize the system by $\mu=40$, $\gamma=1$, $N=4$ and double arrivals, and let us accept the exponentially distributed repair times with mean 0.1. In Fig. 2 we present the time-dependent behaviour of the probability $P\{U(t)=1|U(0)=1\}$ for three different values of the parameter λ*: 10, 20, and 40*, for which the values of the traffic load equal 0.5 (the system is underloaded), 1.0 (the system is critically loaded) and 2.0 (the system is overloaded). Let us note that the stabilization of the transient distribution about the stationary one is the slowest one for the highest value of the traffic load. Moreover, evidently, the stabilizations "occur" on different levels: successive stationary probabilities π_1 (for intensities 10, 20 and 40) equal 0.136631, 0.132278 and 0.07582325, respectively. Similarly, taking λ=40 and $\mu=$ *40, 80 and 160*, and the remaining parameters the same, we obtain (see Fig. 3) similar characterization of the time-dependence of the queue-size distribution on the service speed.

5. DEPENDENCE ON DURATIONS OF FAILURE-FREE AND REPAIR PERIODS

In Fig. 4 we present the time-dependent behaviour of the transient probability $P\{U(t)=4|U(0)=2\}$ for three different working-repair scenarios: *MFFT=1, MRT=0.1* (S1), *MFFT=1, MRT=0.5* (S2), and *MFFT=0.2, MRT=0.2* (S3), where MFFT and MRT stands for mean failure-free time and mean repair time, respectively. The remaining system parameters are the following: $\lambda=\mu=10$, $N=4$, $p_2=1$ (double arrivals).

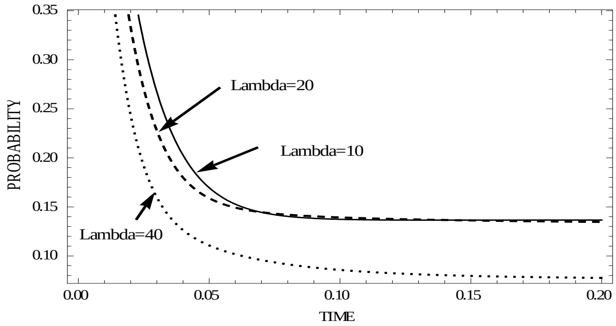

Fig. 2. Probabilities $P\{U(t)=1|U(0)=1\}$ for different values of intensity of arrivals

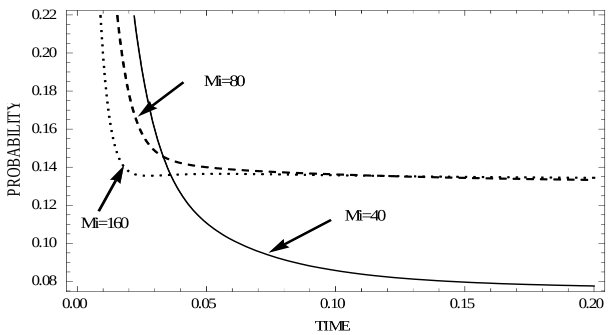

Fig. 3. Probabilities $P\{U(t)=1|U(0)=1\}$ for different values of service speeds

6. STATIONARY QUEUE-SIZE DISTRIBUTION IN A FUNCTION OF BATCH SIZES

Finally, let us study the dependence of the stationary queue-size distribution on the probability distribution of the batch size of jobs. Three different examples, in which we take $p_1=1$ (single arrivals), $p_2=1$ (double arrivals) and $p_1=p_2=0.5$ (single-double arrivals) are presented in Table 1, where the remaining parameters of the system are $\lambda=\mu=30$, $\gamma=1$, *MRT=0.1* (exponential) and *N=4*.

Table 1. Stationary queue-size distribution in a function of the arriving batch structure

Probability /Batch type	Single	Double	Single/double
π_0	0.177112	0.0738664	0.107335
π_1	0.181540	0.0757131	0.110018
π_2	0.189399	0.153312	0.168784
π_3	0.200029	0.236143	0.232307
π_4	0.251920	0.460959	0.381555

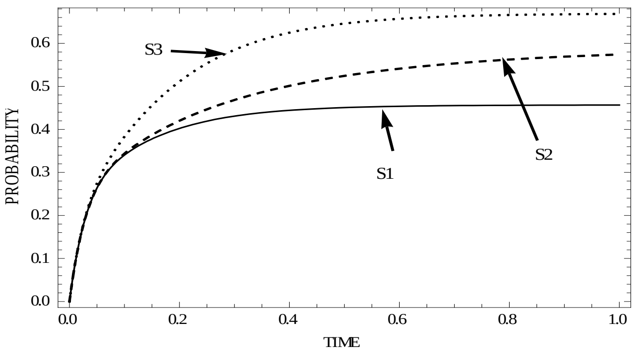

Fig. 4. Probabilities $P\{U(t)=4|U(0)=2\}$ for three different working-repair scenarios

7. CONCLUSIONS

In the article the influence of changing the key operating parameters on time-dependent transient queue-size distribution is studied numerically in the model of manufacturing line with single unreliable machine, batched Poisson arrivals and exponential processing times. In particular, the dependence on the initial buffer state, the intensity of job arrivals, the processing speed and the values of the mean failure-free and repair time is investigated. Moreover, the dependence of the stationary queue-size distribution on the function describing the size of the arriving batch of jobs is analyzed.

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