MODELLING OF FLUCTUATIONS IN THE MAIN BEARING FRAME OF RAILCAR

Ziyoda Mukhamedova[1]

[1] Tashkent Institute of Railway Transport Engineers, Department of Transport Logistics
1, Adilkhodjaev Street, Tashkent 100167, Uzbekistan

Corresponding author: Ziyoda Mukhamedova, ziyoda.mukhamedova@email.com

Abstract: Problems of increasing the reliability and strength of the frames, load-bearing body structure and components of railcards are one of the important tasks among engineers, mechanics, and railway vehicle servicing agencies to manage. Consequently, successful implementation of such measures to enhance and improve the technical aspects of railcar components will improve overall performance of the railway transport. Accordingly, this article covers the use of analytical technique of solutions for flexural and longitudinal fluctuations of the bearing framework of a railcar body frame in the form of an elastic core of variable section with a variable weight, flexural and longitudinal rigidity. The calculation is performed for the modernization of the body frame of emergency and repair rail service car, taking into account the variability of section, mass, longitudinal and bending stiffness along the length to prolong the service life of their useful operation. The introduction of damping subfloor in frame design emergency replacement railcar reduces bending stresses in the frame (10÷12)% and ultimately mathematical model reduces the bending stresses on the main bearing frame.

Key words: railcar, main frame of the body, analytical-numerical method, reliability, strength.

1. INTRODUCTION

In modern foreign patent and scientific literature the problems of increasing the reliability and strength of the frames, load-bearing body structure and components for rail vehicles during their design, operation and modernization are extensively studied (Spryagin *et al.*, 2014; Popp and Schiehlen, 2013). We offer an analytical-numerical method based on the dynamic strength of the bearing body frame of emergency and repair rail service car, assuming a beam-type pattern of its fluctuations with elastic fixing of the ends under harmonic load as it moves along the track with periodic joint roughness.

This article provides a calculation algorithm for the simulation of stress-strain state of load-bearing body frame of emergency and repair rail service car; it gives the results of numerical studies on stress-strain state of bearing body frame structure of emergency and repair rail service car taking into account the variability of section, mass, longitudinal and bending stiffness along its length; it outlines the validity for the choice of diagnostic parameters for the evaluation of dynamic strength, reliability and predictable service life of bearing body frame structure of emergency and repair rail service cars.

Equivalent bearing body frame of emergency and repair rail service car was simulated by an elastic rod with variable cross section, with variable mass, bending and longitudinal stiffness. The difference between the proposed model and the existing ones is an account of the variability of cross section, mass, and the longitudinal and bending stiffness along the length of equivalent beam, which corresponds to the actual conditions of operation (Khromova and Babadjanov, 2005; Khromova *et al.*, 2006). In existing methods of calculation a beam of uniform strength is considered for the simplification, or an approximate calculation is carried out on the model with lumped parameters, excluding elasticity. These approximate models in dynamics may create an error up to (150-200)% of the real strains and stresses. Therefore, in practice, pilot studies are always performed and dynamic correction coefficients are introduced into the calculations of strength and stability.

Turning to the previous studies on this subject area, similar researches with focus on mathematical modelling for repair of defective rail wheels was conducted by researchers from Vilnius Gediminas Technical University, Marijonas Bogdevicius, Rasa Zygiene, Bureika Gintautas and Rimantas Subačius. The research conducted by first two researchers allowed to construct mathematical models for assessing the impact of the uneven railroads and other elements on the structures of the rail car, especially wheels (Bogdevicius and Zygiene, 2015). Whereas, Bureika and Subačius concentrated on mathematical models for calculating bending tensions noted in various elements of the rail car (Bureika and Subačius, 2002). Moreover, numerical modelling by Ioan Sebesan and Dan Baiasu covered the impact of yawing oscillations on body, bogie and wheel elements and allowed for passenger car to be used regularly at the speed of 160 km per hour. As can be noted from these researches, the current article provides similar approach with focus on mathematical modelling of fluctuations in main bearing frame of railcar rather than wheels (Sebesan and Baiasu, 2012).

2. NUMERICAL MODEL OF OSCILLATIONS

For the model proposed here, the parameters of the equivalent load-bearing body frame of the locomotive are taken in the form of variable functions:

- The mass per unit length of the body frame of emergency and repair rail service car (kg/m)

$$m_K(X) = m_O * (a_o + a_1 X + a_2 X^2) \quad (1)$$

- the area of cross section

$$F(X) = F_O * (d_o + d_1 X + d_2 X^2) \quad (2)$$

The length of the main bearing body frame of emergency and repair rail service car is 12.96 meters and the X coordinate varies in the range $0 \le X \le 12,96$ m:

- the reduced moment of inertia of frame section on the axis X_C - I_X (cm^4):

$$I_X(X) = I_O * (b_O + b_1 X + b_2 X^2) \quad (3)$$

- the reduced bending stiffness

$$S_I(X) = E * I_X(X) \quad (4)$$

where $I_X(X)$ is calculated by the Eq.(3).

An assumption is made that the body frame of rail service car is represented in the form of an elastic rod (beam) with constant modulus of material elasticity E = const and the density ρ = const; it has some static initial radius of deflection R. The equations of bending and longitudinal oscillations for this model are taken by analogy (Khromova and Babadjanov, 2005; Khromova *et al*., 2006).

$$m_K(X)\frac{\partial^2 U(X,t)}{\partial t^2} - E\frac{\partial F(X)}{\partial X}\cdot\frac{\partial U(X,t)}{\partial X} - EF(X)\frac{\partial^2 U(X,t)}{\partial X^2} = N_D(X,t) + E\frac{\partial I_X(X)}{\partial X}\cdot\frac{1}{R^2} + 2EI_X(X)\frac{1}{R}\frac{\partial^3 W(X,t)}{\partial X^3} \quad (5)$$

$$m_K(X)\frac{\partial^2 W(X,t)}{\partial t^2} + EI_X(X)\frac{\partial^4 W(X,t)}{\partial X^4} + E\frac{\partial^2 I_X(X)}{\partial X^2}\cdot\frac{\partial^2 W(X,t)}{\partial X^2} = P_D(X,t) + \frac{E}{R}\left[\frac{\partial^2 I_X(X)}{\partial X^2} + 2I_X(X)\cdot\frac{\partial^3 U(X,t)}{\partial X^3}\right] \quad (6)$$

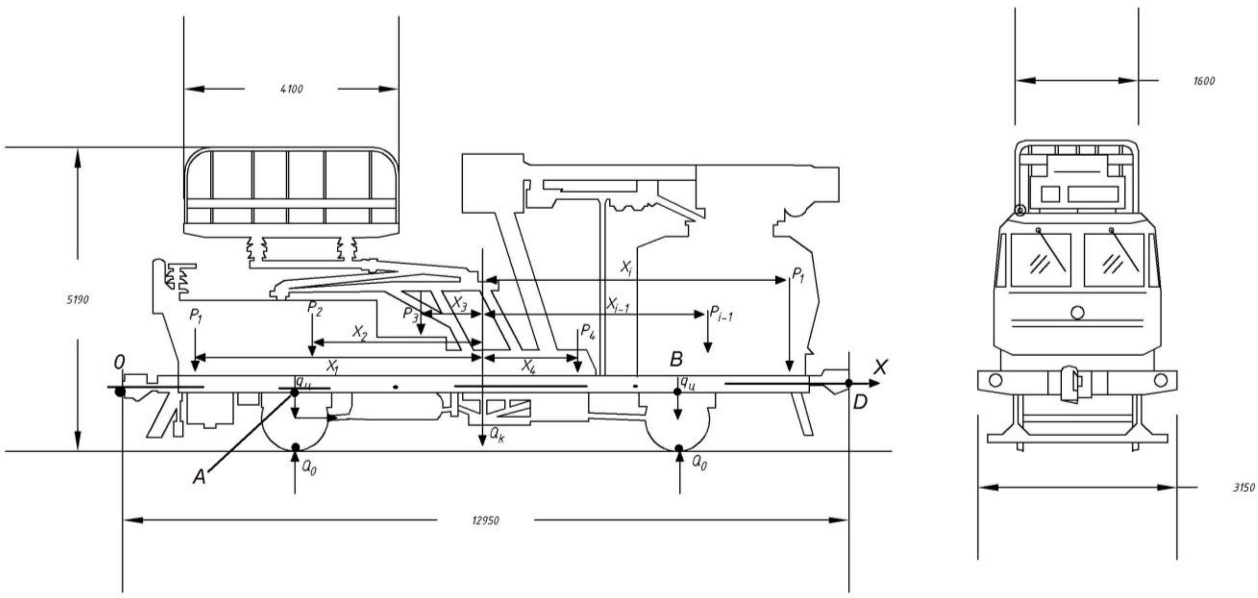

Fig. 1. Design scheme for the equivalent load-bearing frame of the body frame of railcar

To analyze the stress-strain state of equivalent frame of bearing structure of emergency and repair rail service car, the differential equations of bending and longitudinal oscillations of straight rods of variable section are used (considering torsional oscillations relatively small compared to other components) by analogy (Khromova and Babadjanov, 2005; Khromova *et al*., 2006).

After substituting the Eqs. (1)÷(4) and their derivatives in the system of differential Eqs. (5)÷(6) we obtain the nonlinear equations of the form:

$$\left[m_0 \cdot \left(a_0 + a_1 X + a_2 X^2\right)\right]\frac{\partial^2 U(X,t)}{\partial t^2} - E\left[F_0 \cdot \left(d_1 + 2d_2 X\right)\right]\cdot \frac{\partial U(X,t)}{\partial X}$$

$$- E\left[F_0 \cdot \left(d_0 + d_1 X + d_2 X^2\right)\right]\frac{\partial^2 U(X,t)}{\partial X^2} = N_D(X,t) + \qquad (7)$$

$$+ E \cdot \left[I_0 \cdot \left(b_1 + 2b_2 X\right)\right]\cdot \frac{1}{R^2} + 2E \cdot \left[I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right]\cdot \frac{1}{R}\frac{\partial^3 W(X,t)}{\partial X^3}$$

$$\left[m_0 \cdot \left(a_0 + a_1 X + a_2 X^2\right)\right]\cdot \frac{\partial^2 W(X,t)}{\partial t^2} + E \cdot \left[I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right]\cdot \frac{\partial^4 W(X,t)}{\partial X^4} +$$

$$+ E \cdot 2b_2 \cdot I_0 \frac{\partial^2 W(X,t)}{\partial X^2} = P_D(X,t) + \frac{E}{R}\cdot\left[2b_2 I_0 + 2\cdot\left[I_0 \cdot \left(b_0 + b_1 X + b_2 X^2\right)\right]\cdot \frac{\partial^3 U(X,t)}{\partial X^3}\right] \qquad (8)$$

Dividing term by tern each of the Eqs. of the system (7)÷(8) by $m_K(X)$, the entire frame of the body is divided into 120 points (X coordinate varies in the range of $0 \le X \le 12{,}96$ m), for each of the given K-section the coefficients in the Eqs. of the system (7)÷(8) are constant and they could be introduced by iteration method (piecewise linear approximation) into computer solution in the procedure similar to the ones in [3÷4].

After the introduction of notations, we obtain the Eqs. of the form:

$$\frac{\partial^2 U(X,t)}{\partial t^2} - A_{K1}(X)\cdot\frac{\partial U(X,t)}{\partial X} - B_{K1}(X)\frac{\partial^2 U(X.t)}{\partial X^2} = C_{K1}(X)\cdot \sin n\omega t + \qquad (9)$$

$$+ D_{K1}(X) + E_{K1}(X)\cdot\frac{\partial^3 W(X,t)}{\partial X^3}$$

$$\frac{\partial^2 W(X,t)}{\partial t^2} + A_{K2}(X)\cdot\frac{\partial^4 W(X,t)}{\partial X^4} + B_{K2}(X)\cdot\frac{\partial^2 W(X,t)}{\partial X^2} = \qquad (10)$$

$$= C_{K2}(X)\cdot\cos n\omega t + D_{K2}(X) + E_{K2}(X)\cdot\frac{\partial^3 U(X,t)}{\partial X^3}$$

where the following notation are introduced:

- for longitudinal oscillations of the body frame of rail service car – Eq. (9)

$$A_{K1}(X) = \frac{EF_0\left(d_1 + 2d_2 X\right)}{m_K(X)} \;;\quad B_{K1}(X) = \frac{EF_0\left(d_0 + d_1 X + d_2 X^2\right)}{m_K(X)}$$

$$C_{K1}(X) = \frac{N_{\ddot{A}K}(X)}{m_K(X)}\,,\; N_{\ddot{A}K}(X) = N_{\ddot{A}\cdot n}\sin\frac{(2n-1)\cdot\pi\cdot X}{2\ell_0}$$

Here the horizontal external dynamic load is taken in the form:

$$N_{DK}(X,t) = N_{D\cdot n}\sin\frac{(2n-1)\cdot\pi\cdot X}{2\ell_0}\cdot\sin n\omega t \qquad (11)$$

where $n = 1,2,3\ldots5$ – is a number of harmonics, N_{Dn} – is taken according to experimental data obtained, depending on different modes of loading:

$$D_{K1}(X) = \frac{E\cdot\left(I_0\cdot\left(b_1 + 2b_2 X\right)\right)}{m_K(X)}\cdot\frac{1}{R^2} \;;\quad E_{K1}(X) = \frac{2E\cdot\left(I_0\cdot\left(b_0 + b_1 X + b_2 X^2\right)\right)}{m_K(X)}\cdot\frac{1}{R}$$

- for bending (transverse) oscillations of the body frame of rail service car – Eq. (10)

$$A_{K2}(X)=\frac{E\cdot\left(I_0\cdot\left(b_0+b_1X+b_2X^2\right)\right)}{m_K(X)};\qquad B_{K2}(X)=\frac{2E\cdot\left(I_0\cdot b_2\right)}{m_K(X)};$$

$$C_{K2}(X)=\frac{P_{ÄK}(X)}{m_K(X)},\quad P_{DK}(X)=P_{D\cdot n}\sin\frac{\pi\cdot n\cdot X}{\ell_0}$$

Here the vertical external dynamic load is taken in the form:

$$P_{DK}(X,t)=P_{D\cdot n}\sin\frac{n\cdot\pi\cdot X}{\ell_0}\cdot\cos n\omega t \qquad (12)$$

where n = 1,2,3…5 – is a number of harmonics, P_{Dn} – – is taken according to experimental data obtained, depending on different modes of loading.

$$D_{K2}(X)=\frac{E\cdot\left(2I_0\cdot b_2\right)}{R\cdot m_K(X)};\quad E_{K2}(X)=\frac{2E\cdot\left(I_0\cdot\left(b_0+b_1X+b_2X^2\right)\right)}{m_K(X)}\cdot\frac{1}{R}.$$

The solution of the system (7) ÷ (8) is performed with the linearization by Simpson's method, then Fourier method is applied to the differential equations with constant coefficients with further application of operational Laplace transform in time; numerical studies are carried out by the methods of piecewise linear approximation and boundary elements method, similar to the procedures given in studies of Khromova and Babadjanov (Khromova and Babadjanov, 2005; Khromova *et al.*, 2006) in Mathcad 14 programming environment. Initial conditions are taken as zero ones, and the boundary conditions - in the form of elastic fixing of the ends. Thus, it is possible to find a general solution of differential Eqs. of bending and longitudinal oscillations of the body frame of emergency and repair rail service car (9) and (10) in the form:

$$W(X,t)=\sum_{k=1}^{\infty}W(X)*\{\frac{C_{K2}}{W(X)}\cdot\frac{\cos n\omega t-\cos\lambda_{2n}t}{\lambda_{2n}^2-(n\omega)^2}+W_0\cdot\cos\lambda_{2n}t+$$
$$+[\frac{D_{K2}}{W(X)}+V_{ÏÏ}\ ]*\frac{1}{\lambda_{2n}}\cdot\sin\lambda_{2n}t\} \qquad (13)$$

$$U(t)=\frac{C_{K1}}{U(X)}\cdot\frac{n\omega\cdot\sin\lambda_{1n}t-\lambda_{1n}\sin n\omega t}{n\omega\cdot\lambda_{1n}\cdot\left(\lambda_{1n}^2-(n\omega)^2\right)}+U_0\cdot\cos\lambda_{1n}t+$$
$$+[\frac{D_{K1}}{U(X)}+V_{Ï}\ ]*\frac{1}{\lambda_{1n}}\cdot\sin\lambda_{1n}t+\frac{\hat{O}(X)}{U(X)}*\{\frac{C_{K2}}{W(X)}*W_1(t)+$$
$$+W_0\cdot\frac{\cos\lambda_{2n}t-\cos\lambda_{1n}t}{\lambda_{1n}^2-\lambda_{2n}^2}+\frac{D_{K2}}{W(X)}*\frac{\sin\lambda_{2n}t-\sin\lambda_{1n}t}{\lambda_{1n}^2-\lambda_{2n}^2}+ \qquad (14)$$
$$+V_{ÏÏ}\ \cdot\frac{\sin\lambda_{2n}t-\sin\lambda_{1n}t}{\lambda_{1n}^2-\lambda_{2n}^2}\}$$

where

$$W_1(t)=\frac{\cos\lambda_{1n}t}{\left(\lambda_{1n}^2-(n\omega)^2\right)\cdot\left(\lambda_{2n}^2-\lambda_{1n}^2\right)}-\frac{\cos n\omega t}{\left(\lambda_{1n}^2-(n\omega)^2\right)\cdot\left(\lambda_{2n}^2-(n\omega)^2\right)}-$$
$$-\frac{\cos\lambda_{2n}t}{\left(\lambda_{2n}^2-(n\omega)^2\right)\cdot\left(\lambda_{2n}^2-\lambda_{1n}^2\right)} \qquad (15)$$

Thus, as a result of using the method of iterations and piecewise linear approximation we have managed to obtain an analytical and numerical solution for the analysis of joint bending and longitudinal oscillations of the bearing body frame of emergency and repair rail service car in the form of a model of an elastic rod of variable cross section, mass, bending and longitudinal stiffness as it moves along the track with periodic joint roughness.

Table 1. Results of experimental simulation for measurement of vibrations: with damping and without damping equipment

Checkpoint measurements of vibrations and stresses	The low-frequency component of the acceleration, [Hz]		The maximum amplitude of vibration acceleration, [m/s^2]		The longitudinal tension (in the center of the frame), [MPa]		Bending stress (in the center), [MPa]	
	Experiment	Theory	Experiment	Theory	Experiment	Theory	Experiment	Theory
Frame body control (including damping subfloor)	2.59	2.64	14.06	-	3.2	3.1	28	29.1
Frame body control (standard design)	2.07	2.17	15.2	-	3.3	3.2	31	30.7

In order to better understand and make thorough analysis and conclusions, simulation of the mathematical model was carried out using testing railcars with simulation workplace. The idea behind the experiment was to install in the frame control unit so called damping subfloor element. The results of the simulation experiment are summarized in Table 1. As can be observed from Table 1, experimental data received from simulation is to the greatest extent in accordance with calculated mathematical model and very small deviation. Accordingly, the total stress-strain state with the introduction of the damping subfloor in the frame body structure of railcars decreased by about 11 ÷ 15%, depending on the loading conditions that will facilitate the operation of the extension of the useful life. The total dynamic voltage does not exceed the tensile strength in the experiment ranged from 15.3 MPa to 41.23 MPa.

Furthermore, frequency analysis performed by the program in Mathcad 14 programming environment, showed that the frequencies of natural vibrations vary in harmonics n = 1, 2, 3…5 as follows (see Figures 2, 3):

- under longitudinal vibrations of the system with the introduction of damping bottom covering, the frequency of natural vibrations of modernized frame of electric locomotive λ_{1mn} increases compared to a standard one λ_{1n} (for example, at n = 5 the frequency is 0.587 and 0.602 Hz/m, respectively) (Table 2, Figure 2).

Table 2. Change of natural vibrations of locomotive frame in harmonics (with or without damping bottom covering) under longitudinal vibrations

n	a_n	λ_{1n}	λ_{1mn}
1	0.087	0.065	0.067
2	0.262	0.196	0.201
3	0.436	0.326	0.334
4	0.611	0.457	0.468
5	0.785	0.587	0.602

- under bending vibrations of the system with the introduction of damping bottom covering the frequency of natural vibrations of modernized frame of electric locomotive λ_{2n} drops compared to a standard one $\lambda_2{}^0{}_n$ (for example, at n = 5, it is reduced from 1.321 Hz to 1.253 Hz, respectively), (see Figure 3, Table 3).

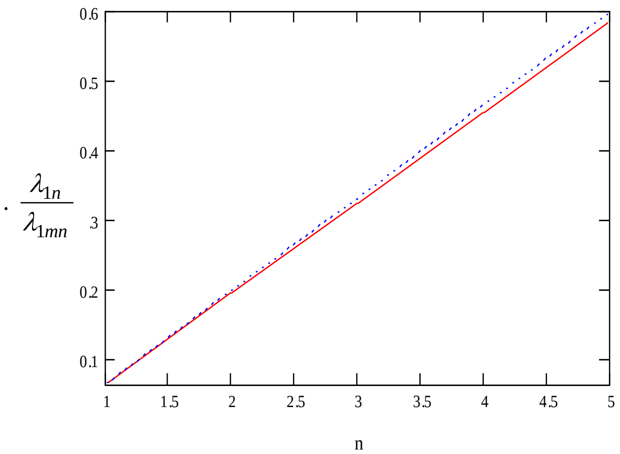

Fig. 2. Diagram of the change in eigenfrequency under longitudinal vibrations (in harmonics) for a standard frame λ_{1n} and a modernized one (with damping bottom covering) λ_{1mn}.

Table 3. Change of natural vibrations of locomotive frame in harmonics (with or without damping bottom covering) under bending vibrations

n	λ_{20n}	λ_{2n}	λ_{1mn}
1	0.44	0.418	0.067
2	0.661	0.627	0.201
3	0.881	0.835	0.334
4	1.101	1.044	0.468
5	1.321	1.253	0.602

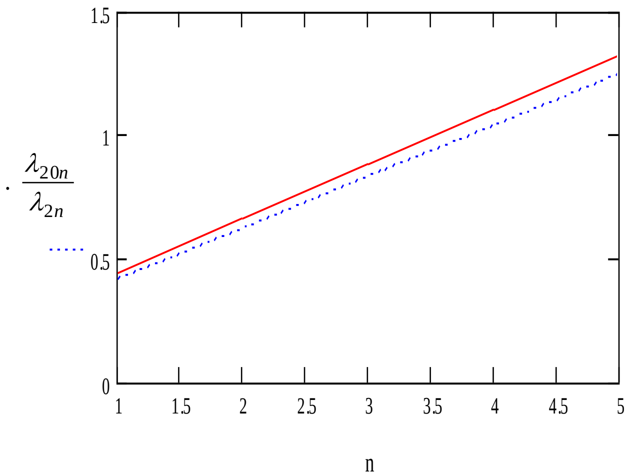

Fig. 3. Diagram of changes in Eigen frequency under bending vibrations (in harmonics) for a standard frame $\lambda_2{}^0{}_n$ and a modernized one (with damping bottom covering) λ_{2n} for the first and the third sections of the frame of emergency repair railcar

Hence, the results of both mathematical model based on experiment (simulation) is in line with proposed improvements for the railcars.

3. CONCLUSIONS

On the basis of numerical studies and comparative analysis with experiment (simulation) we have stated the following quality patterns:

Bending stresses appearing in the center of the length of the body frame of emergency and repair rail service car at speeds up to 50 km/h, as it moves along the track with periodic roughness, do not exceed the ultimate strength of the material, and in average range from 15 to 40 MPa depending on loading modes (the rate of motion).

Longitudinal stresses appearing in the center of the length of the body frame of emergency and repair rail service car at speeds up to 50 km/h, as it moves along the track with periodic roughness, are about 20 ÷ 25% of the bending stresses (from 3 to 10.4 MPa). They reach their maximum values at breakaway and braking modes.

The introduction of damping subfloor in frame design emergency replacement railcar reduces bending stresses in the frame 10 ÷ 12%, depending on the speed (respectively from 31 MPa to 28 MPa at a speed of 40 km / h - 11.07%).

Accordingly, the use of mathematical modelling in modernization and extension of useful life of railcars is highly applicable given the importance of low cost maintenance and use of railway resources effectively. The results of the simulation and mathematical modelling will be implemented in real life conditions and will be compared with data on mathematical calculations and simulation.

4. REFERENCES

1. Spiryagin M., Cole C., Sun Y.Q., McClanachan M., Spiryagin V. & McSweeney T., (2014). *Design and Simulation of Rail Vehicles*. Ground Vehicle Engineering series, pp. 337, CRC Press, Boca Raton.
2. Popp K. & Schiehlen W., (2013). *System Dynamics and Long-Term Behaviour of Railway Vehicles, Track and Subgrade*, pp. 488, Springer Science and Business Media, Heidelberg.
3. Khromova G.A., Babadjanov A.A., (2005). *Development of Analytical and Numerical Calculation Method to Dynamic Strength of the Bearing Body Frame of Governing Electric Locomotive*, Proceedings of the 6th International Scientific and Technical Conference "Train Operation Safety", pp. 87-89, MIIT, Moscow.
4. Khromova G.A., Babadjanov A.A., Zakirov Sh.A., (2006). *Development of Analytical and Numerical Calculation Method to Dynamic Strength of the Bearing Body Frame of Governing Electric Locomotive*, Journal "Industrial Transport of Kazakhstan", 3(9), 14-18.
5. Bogdevicius M. & Zygiene R., (2015). *Simulation of dynamic processes of rail vehicle and rail with irregularities*, Journal of KONES Powertrain and Transport, 21(2), 21-26.
6. Bureika G. & Subačius R., (2002). *Mathematical model of dynamic interaction between wheel-set and rail track*, Transport, XVII (2), 46-51.
7. Sebesan I. & Baiasu D., (2012). *Mathematical model for the study of the lateral oscillations of the railway vehicle*. Scientific Bulletin Series D: Mechanical Engineering. Politehnica University of Bucharest, 74(2), 51-66.

Received: February 10, 2016 / Accepted: December 10, 2016 / Paper available online: December 20, 2016