KINEMATICAL METHOD FOR RACK-GEAR TOOL'S PROFILING, IN CATIA DESIGN ENVIRONMENT

Silviu Berbinschi [1], Virgil Teodor[1] & Nicolae Oancea[1]

[1] Technical University "Dunarea de Jos" of Galati, Manufacturing Science and Engineering Department, Domneasca Street, No. 111, 800201 Galati, Romania

Corresponding author: Nicolae Oancea, nicolae.oancea@ugal.ro

Abstract: In this paper, is proposed a methodology to determine the rack-gear tool's profiling, reciprocally enwrapping with an ordered profile curl, associated with a circular centrode, using the capabilities of CATIA software, by making a kinematical entity which reproduce the rolling movement of centrodes associated with tool and blank. Is presented an application of the general algorithm regarding the rack tool's profiling. They are presented the successive steps of this process and, also, numerical examples regarding the crossing profile of the rack-gear tool. The method quality is proofed by comparing the obtained results with results obtained for the same problem, using one of the analytical methods, the Willis method, and software developed in Java.

Key words: tool's profiling, kinematical method, CATIA

1. INTRODUCTION

The profiling of tools which generate by enveloping with rolling method – rack-gear tool and gear shaped cutter – may be realized, as is known by more methods: analytical methods, based on fundamentals theorem of surfaces enwrapping (1st Olivier theorem, Gohman theorem, method of normals, Willis [1], [2]); complementary analytical methods ("minimum distance" method, method of "substitutive circles family", "the in-plane generating trajectory" method) [3], [4]; grapho-analitical methods [5]; graphical methods, which use the capabilities of CAD software [6], [7].

2. GENERATING KINEMATICS

In principle, the problem for profiling the rack-gear tool reciprocally enveloping with an ordered profiles curl, associated with a circular centrode, assume to follow the generating process kinematics, see figure 1.
The two rolling centrodes, C_1 associated with the ordered profiles curl to be generated and C_2 rectilinear, associated with the rack tool's space, are in rolling, so, is permanently follow the condition

$$\lambda = R_{rp} \cdot \varphi \qquad (1)$$

where: λ is the linear velocity in the translating motion of C_1 centrode; $R_{rp} \cdot \varphi$ — the velocity in point P, the gearing pole, on the centrode C_1, in the

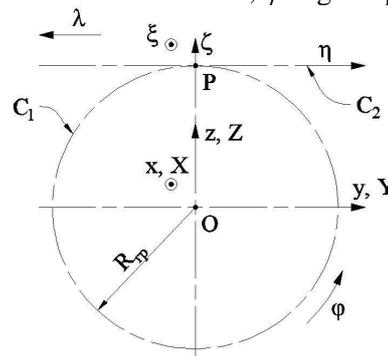revolving motion around Z axis; φ angular parameter.

Fig. 1. A couple of rolling centrodes; generating kinematics

The principled kinematics of the rolling process between the two centrodes, C_1 and C_2, tangents in point P — the gearing pole — assume that the speed of points belongs to the two centrodes, temporally found in point P, to be equals.
For the equations of the surfaces profile, belongs to family to be generated

$$\Sigma = \begin{pmatrix} 0 \\ Y(u) \\ Z(u) \end{pmatrix}, \qquad (2)$$

with u variable parameter, is generated, in the $\xi\eta\zeta$ reference system, the profile family:

$$(\Sigma)_\varphi \begin{cases} \xi(u,\varphi) = 0; \\ \eta(u,\varphi) = Y(u)\cos\varphi - Z(u)\sin\varphi + R_{rp} \cdot \varphi; \\ \zeta(u,\varphi) = Y(u)\sin\varphi + Z(u)\cos\varphi - R_{rp}, \end{cases} \quad (3)$$

of which enveloping is the profile of rack tool.

2.1 The kinematics method in the CATIA design environment

Is proposed a new solution of the rack tool's profiling, using the capabilities of CATIA software, by making a kinematics entity which reproduce the rolling movement of centrodes: circle with radius R_{rp}, associated with the profile's curl and straight line, which is associated with the space of the rack tool.
The proposed solution is based on the facilities of the

Part environment (*Part Environment*), in which is synthesized the elements of a mechanism, capable to simulate the enwrapping condition, in this case the condition of normals. These elements, created in *Part* environment, are introduced in a file of the *Assembly* environment, assuring the positioning of the mechanism elements in the start position, and then, in the *DMU Kinematics* environment (*Digital Mock Up*) to establish the predefined kinematics couples.

The mechanism movement is making by command *Simulation*, establishing a number of intermediary positions *Shots*, creating with *Replay* command, a movie of the successive positions of the mechanism.

With command *Trace* is traced the trajectory of any point from an element of the mechanism regarding any another element of this, including the global reference system, determining in this way, the gearing line between the profile to be generated and the rack tool's profile.

These trajectories represent curves on type *Spline*, obtained by successive points obtained by mechanism rolling. The coordinates of these points may be extracted as text files, see figure 2.
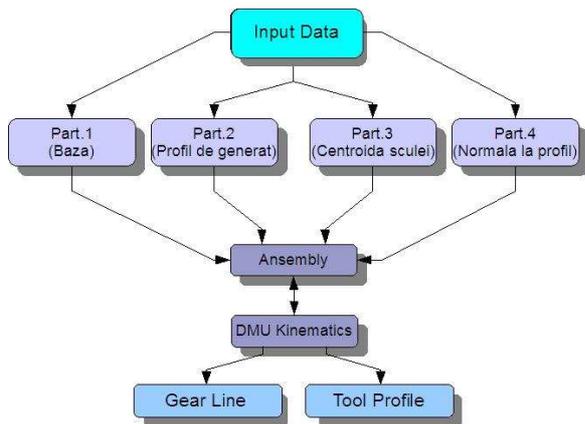

Fig. 2. Algorithm for generating in the CATIA design environment

3. APPLICATION: RACK-GEAR TOOL FOR A POLYGONAL SHAFT

To obtain the tool's profile for a variety of polygonal shafts, means for various length of the polygon side, are need two stages.

The first step is to indicate the side length and the diameter of the rolling circle, as input data. This may be making by introducing in a text or Excel file the values, following that in CATIA to be modified automatically the entire mechanism, according the new input data. In the same stage, is created the mechanism, the rolling of this and the tool's profiling. The export of the coordinates of points on profile may be making in *txt* format or in *Excel* format. The second stage, consist in the partially remaking of the mechanism's couples, the rolling of mechanism in order to obtain the new tool's profile, for the previously modified values and the export in a

new file of the points belongs to profile.

The mechanism elements are created one at a time, in the *Part* environment, and are inserted in an assembly file created in the *Assembly* environment, named *GenCremaliera*, figure 3.

Fig. 3. The CATIA tree representing the assembly file, with all the *Part* files

The *Baza* file contains the fixed part of the mechanism and represents more lines which will fix the other elements, figure 4.
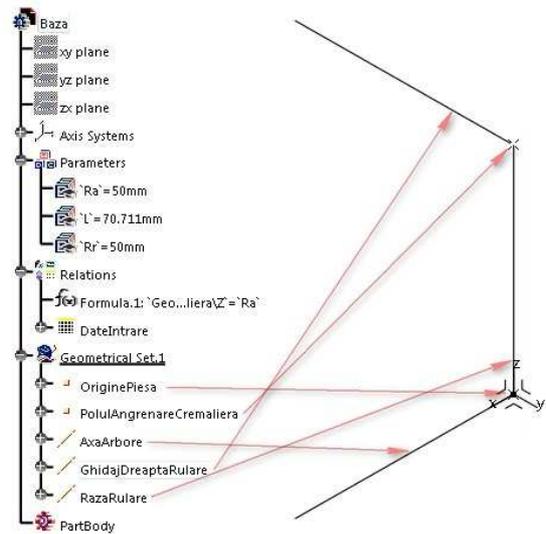

Fig. 4. The *Baza* file

The *Piesa* file contains the geometrical elements of the profile to be generated, figure 5.
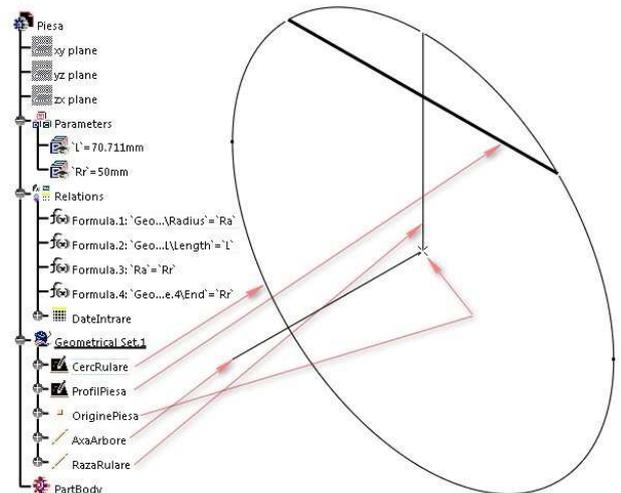

Fig. 5. The *Piesa* file

Cremaliera is the *Part* file which contains the rolling straight line, with the length of a circular pitch of the curl to be generated, see figure 6.
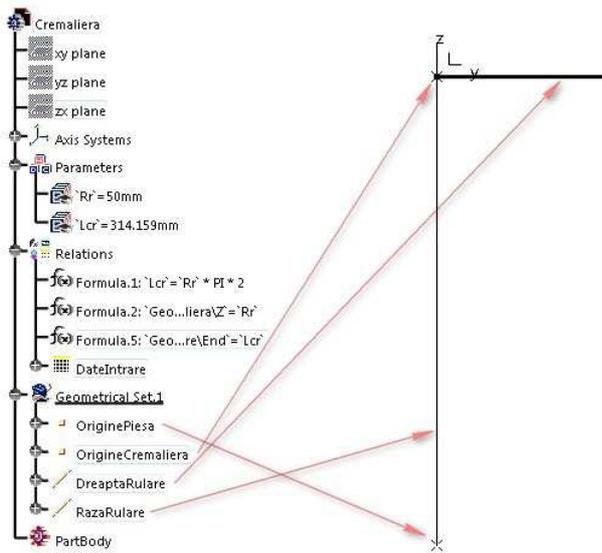
Fig. 6. The *Cremaliera* file

In table 1, are presented the constraints and the links between the mechanism elements.
The *Tachet* file, see figure 7, contains two lines: *Tangenta* which is tangent to the piece's profile and *Normala*, in contact point crossing the gear pole.

Fig. 7. The *Tachet* file

In the assembly file *GenCremaliera* are introduced all of the elements, with the command *Insert Existing Component*.
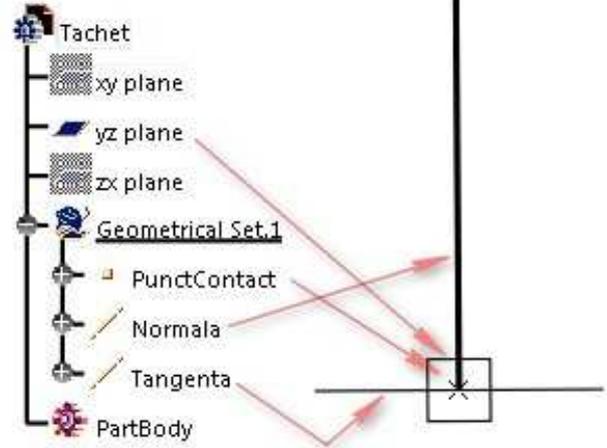
Table 1. Constraints in the assembly environment

Crt. no.	Constraint type	Mechanism elements	
1	*Fix*	*Baza*	-
2	*Coincidence*	Plane YZ (of the element *Baza*), fig. 4	Plane YZ (of the element *Piesa*), fig. 5
3	*Coincidence*	Plane YZ (of the element *Baza*), fig. 4	Plane YZ (of the element *Cremaliera*), fig. 6
4	*Coincidence*	Plane YZ (of the element *Baza*), fig. 4	Plane YZ (of the element *Tachet*), fig. 7
5	*Coincidence*	Axis (of the element *Baza*), fig. 4	Axis (of the element *Piesa*), fig. 5
6	*Coincidence*	*GhidajCremaliera* (of the element *Baza*), fig.4	*DreaptaRulare* (of the element *Cremaliera*), fig. 6
7	*Coincidence*	*PolulAngrenare* (of the element *Baza*), fig.4	*Normala* (of the element *Tachet*), fig. 7
8	*Coincidence*	*PunctContact* (of the element *Tachet*), fig.7	An end point (vertex) of profile from *Piesa* file, fig. 5
9	*Coincidence*	*Tangenta* (of the element *Tachet*), fig. 7	Profile side *L* (*Piesa*), fig. 5

Table 2. Joints in the DMU Kinematics environments

Crt. no.	Joints type	Mechanism elements	
1	*Fix*	Baza	-
2	*Revolute*	*Axa* (of the element *Baza*), fig.4	*Axa* (of the element *Piesa*), fig.5
3	*Prismatic*	*GhidajCremaliera* (of the element Baza), fig.4	*DreaptaRulare* (of the element *Cremaliera*), fig.6
4	*Rack*	YZ plane (of the element *Baza*), fig.4	YZ plane (of the element *Tachet*), fig.7
5	*Prismatic*	*Tangenta* (of the element *Tachet*), fig.7	Profile side *L* (of the element *Piesa*), fig.5
6	*PointCurve*	*PolulAngrenarii* (of the element *Baza*), fig.4	*Normala* (of the element *Tachet*), fig.7
7	*PointCurve*	*PunctContact* (of the element *Tachet*), fig.7	Profile side *L* (of the element *Piesa*), fig.5
8	*Rigid*	*SculaHexagon*	-

After these constraints, are created the joints mechanism, see table 2.

In the *DMU Kinematics* environment, CATIA allow to create more mechanism, with the same assembly elements as in the *Assembly* environment with the joints from the *DMU Kinematics* toolbar.

In figure 8, are represented some of the shootings of the mechanism rolling.

a).   b).

c).   d).
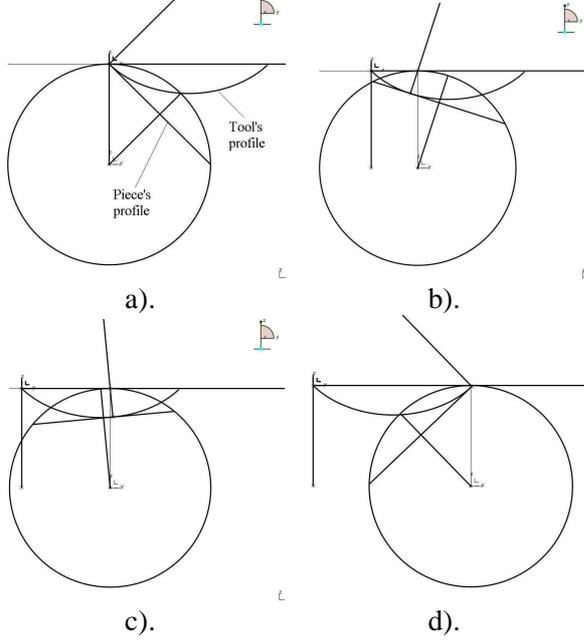
Fig. 8. Successive positions (a-d) of the mechanism in the tool's profile generating process

4. THE POINT'S COORDINATES EXTRACTION

After the determination of the tool's profile, the point's coordinates may be exported in a text file, using command *Design Table*, from the *Part* menu of the file *SculaPatrat.part*, previously generated. This command will create a text file, *PuncteProfil.txt*.

The points on this file may be edited with any spreadsheet program, as *OpenOffice*, *Excel* etc., for a easy communication with another software or numerical controlled machines, which will machine the rack-gear tool's profile.

In the second stage, if the user modifies the *DateIntrare.txt* file, saving the modification of the input parameters, the CATIA software detect this thing and warning by the *Update* message.

After this message the user have to update the assembly file and the kinematics file in order to remake the kinematics couples which may be modified.

5. METHOD QUALITY VERIFICATION

It is proposed in following the verification of this method by comparing the numerical results obtained with graphical method regarding the results obtained by one of the analytical methods: Gohman method; "normals" method; "minimum distance" method etc.

5.1. The Generating of Straight lined profiles

For the polygonal shaft is defined the analytical form of the profile to be generated, see figure 9:

$$\Sigma \begin{vmatrix} X = 0; \\ Y = u; \\ Z = a, \end{vmatrix} \tag{4}$$

with u variable parameter, measured along the profile and a constant value depending the profile form.
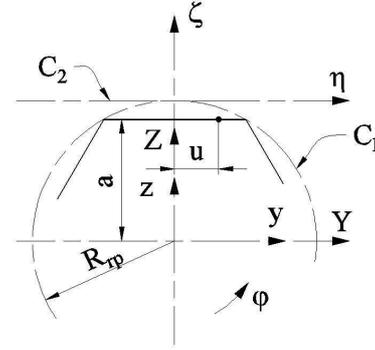
Fig. 9 – Straight lined profile

The Σ profile family is on form:

$$(\Sigma)_\varphi \begin{vmatrix} \xi = 0; \\ \eta = u\cos\varphi - a\sin\varphi + R_{rp}\varphi; \\ \zeta = u\sin\varphi + a\cos\varphi - R_{rp}, \end{vmatrix} \tag{5}$$

with φ variable parameter.

The specifically enveloping condition, in the "in-plane generation trajectory" methods is:

$$\frac{\cos\varphi}{\sin\varphi} = \frac{-u\sin\varphi - a\cos\varphi + R_{rp}}{u\cos\varphi - a\sin\varphi} \tag{6}$$

or

$$u = R_{rp}\sin\varphi \tag{7}$$

The equations assembly (5) and (7), for u variable between limits $u_{min} = -0.5 \cdot R_{rp}$; $u_{max} = 0.5 \cdot R_{rp}$, for a hexagonal shaft, represent the rack-gear tool's profile reciprocally enwrapping with the shaft's profile.

In table 3, are presented the tool's profile coordinates determined by the analytical method [6], [7], [16], [17].

In figure 9, are presented the profiles form and the errors obtained at profiling, regarding the coordinates transformation:

$$\zeta_{CATIA} = -\xi_{JAVA};$$
$$\eta_{CATIA} = \eta_{JAVA} + 26.1799. \tag{8}$$

Fig. 9 – Errors obtained

Table 3. Points coordinates on tool's profile—normal method, in Java program

Crt. no.	ξ [mm]	η [mm]
1	0.000	-26.180
2	0.025	-26.137
3	0.100	-26.006
4	0.125	-25.962
5	0.175	-25.875
6	0.200	-25.831
7	0.249	-25.743
8	0.274	-25.699
9	0.323	-25.611
⋮	⋮	⋮
244	6.691	-0.938
245	6.694	-0.768
246	6.695	-0.711
247	6.696	-0.540
248	6.697	-0.483
249	6.698	-0.256
250	6.699	-0.199
251	6.699	-0.028
252	6.699	0.028
253	6.699	0.199
254	6.698	0.256
255	6.697	0.483
256	6.696	0.540
⋮	⋮	⋮
492	0.323	25.611
493	0.274	25.699
494	0.249	25.743
495	0.200	25.831
496	0.175	25.875
497	0.125	25.962
498	0.100	26.006
499	0.025	26.137
500	0.000	26.180

The others elements and commands remain unchanged.

Fig. 10. M.G.M.C. for circle's arc

Fig. 11. Profile's geometry of a sprocket tooth flank

5.2. The Generating of Circle's Arc Profile

The rack-gear tool's profiling for a profile composed from circles arc, as sprocket, impose to create in DMU Kinematics a specifically virtual mechanism for generation by enwrapping of a circle's arc, see figure 10.

For the tool's profile determination, which generates the flank of a sprocket tooth, the algorithm is the same as this for the previous profile. The piece profile will contain two drafts with the flank profile, see figure 11.

The particularity regarding the virtual mechanism for generating in CATIA environment (M.G.M.C.) for straight line segments consist in fact the translation couple between Sledge and Piece is replaced with a rotation couple around an axis which cross the centre arc representing the profile.

After the creation of the file *Pieces* in DMU Kinematics environment are created the mechanism couples specifically for curved profiles. For this, the couple *Prismatic* from the straight lined profiles are replaced with *Revolute* couples for axis of circle's arc which represent the profile.

Fig. 12. Sprocket—assembly—Piece's file

This couple type allow to rotate the element *Sledge*, by the axis from the circle's arcs (*AxaC1* and *AxaC2*), around similarly axis from the file *Piesa*, while the couple *PointCurve*, between *ContactPoint* from the file *Sledge*, with *PieceProfile* from file *Piece* remain as leader element.

In the DMU Kinematics environment will be realised two mechanisms, see figure 12, for each arc of the composed profile, see table 4.

After the creation of the mechanism, of the simulation and after the record of the kinematics, with command *Trace*, is draw the tool's profile.

In figure 13 and table 5, is presented the rack-gear tool's profile for the sprocket with characteristics: Rr=Re=55 mm; z=12 teeth; R2=25 mm; R1=5 mm; β=55°; Rd=45 mm.

Table 4. Kinematics couples for the sprocket profile

Crt. no.	Joint type	Mechanism elements	
1	Fix	Base	-
2	Revolute	Axis (*Base*)	Axis(*Piece*)
3	Prismatic	Rack-gear guide(*Base*)	Rolling line (*Rack-gear*)
4	RollCurve	Rolling circle(*Piece*)	Rolling circle (*Rack-gear*)
5	Revolute	AxaC1, AxaC2 (*Base*)	AxaC1,AxaC2 (*Piece*)
6	PointCurve	Gearing pole(*Base*)	Normal(*Sledge*)
7	PointCurve	Contact point(*Sledge*)	Profile's side L (*Piece*)
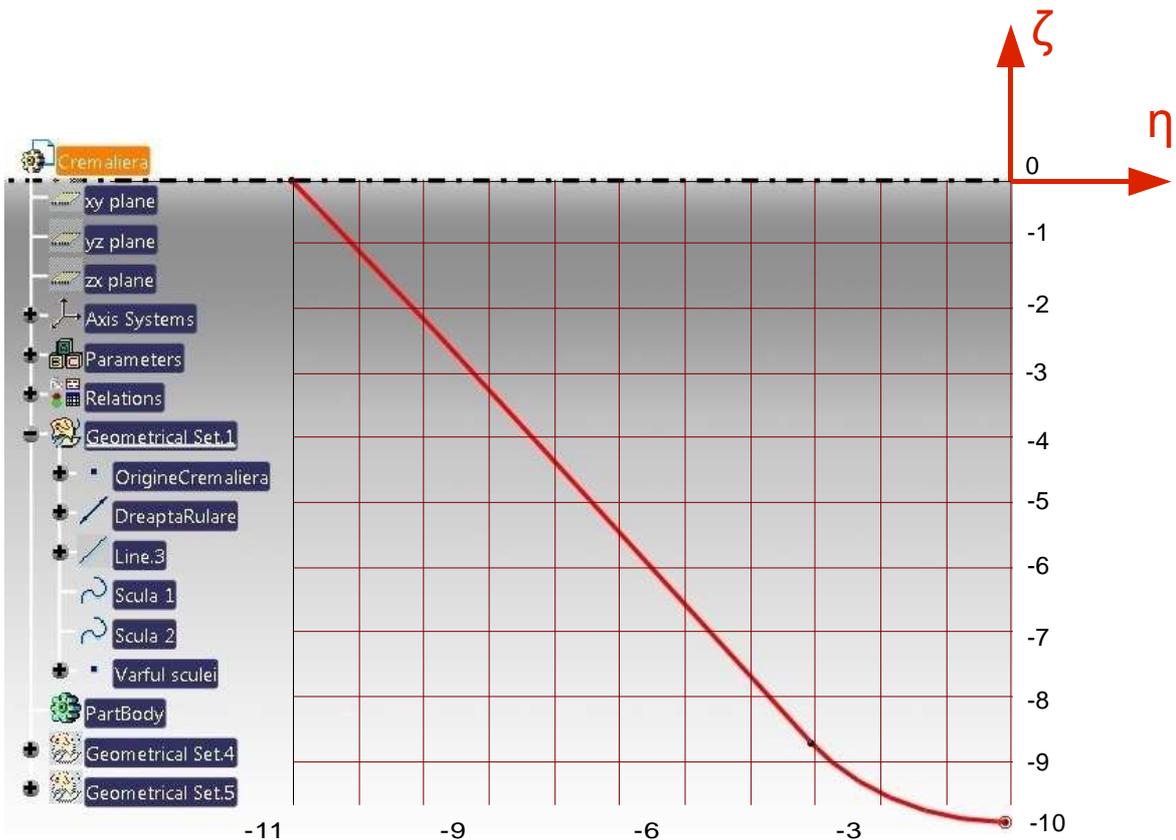
Fig. 13. Rack-gear profile for sprocket

Analytical solution

According to figure 11, are defined the parametrical equations for the two circle's arc, which constitute the sprocket flank:

$$AB \begin{vmatrix} Y = -R_1 \sin\theta_1; \\ Z = R_d - R_1 \cos\theta_1, \end{vmatrix} \quad \theta \in [0, \beta] \quad (9)$$

and

$$BC \begin{vmatrix} Y = -(R_1 + R_2)\sin\beta + R_2 \sin(\beta - \theta_2); \\ Z = R_d - (R_1 + R_2)\cos\beta + R_2 \cos(\beta - \theta_2). \end{vmatrix} \quad (10)$$

In this way, from (3), may be deduced the profile's family equations in the relative motion of the two centrodes C_1 and C_2, see figure 1:

Table 6. Coordinates of points on the rack-gear profile

Crt. no.	$\eta[mm]$	$\zeta[mm]$	Crt. no.	$\eta[mm]$	$\zeta[mm]$
For first zone			*For second zone*		
1	-2.9610	6.7801	1	0.5979	0.3265
8	-2.6400	6.2515	8	0.7680	0.0442
15	-2.3974	5.8403	15	0.9655	-0.2299
22	-2.1598	5.4285	22	1.1867	-0.4907
29	-1.9260	5.0149	29	1.4289	-0.7350
36	-1.6946	4.5982	36	1.6897	-0.9610
43	-1.4641	4.1767	43	1.9671	-1.1672
50	-1.2327	3.7485	50	2.2592	-1.3523
57	-0.9985	3.3111	57	2.5641	-1.5155
64	-0.7594	2.8619	64	2.8802	-1.6559
71	-0.5130	2.3978	71	3.2055	-1.7730
78	-0.2569	1.9157	78	3.5383	-1.8662
85	0.0117	1.4121	85	3.8770	-1.9351
92	0.2956	0.8835	92	4.2197	-1.9795
100	0.5979	0.3265	100	4.6634	-2.0000

$$(AB)_{\varphi_1} \begin{vmatrix} \eta = -R_1 \sin(\theta_1 - \varphi_1) - R_d \sin\varphi_1 + R_{rp}\varphi_1; \\ \zeta = R_1 \cos(\theta_1 - \varphi_1) + R_d \cos\varphi_1 - R_{rp}. \end{vmatrix} \quad (11)$$

and

$$(BC)_{\varphi_1} \begin{vmatrix} \eta = -(R_1 + R_2)\sin(\beta_1 - \varphi_1) + \\ +R_2 \sin(\beta - \theta_2 - \varphi_1) - R_d \sin\varphi_1 + R_{rp}\varphi_1 \\ \zeta = -(R_1 + R_2)\cos(\beta_1 - \varphi_1) + \\ +R_2 \cos(\beta - \theta_2 - \varphi_1) + R_d \cos\varphi_1 - R_{rp}. \end{vmatrix} \quad (12)$$

The specifically enwrapping conditions, determined by the in-plane generating trajectories (6) are:
- for $(AB)_{\varphi_1}$ family,

$$\varphi_1 = \theta_1 - arcsin\left(\frac{R_d}{R_{rp}}\sin\theta_1\right); \quad (13)$$

- for $(BC)_{\varphi_1}$ family,

$$\varphi_1 = \beta_1 - \theta_2 - $$
$$- arcsin\left[\frac{R_d \sin(\beta - \theta_2) + (R_1 + R_2)\sin\theta_2}{R_{rp}}\right] \quad (14)$$

The variation limits of parameters,

$$0 \le \theta_1 \le \beta, \ \beta = 55° \quad (15)$$

and

$$0 \le \theta_2 \le \theta_{2max}, \quad (16)$$

θ_2 is determined, from the intersection condition between the arc with radius R_2 with the external circle of wheel ($R_c=R_{rp}=50$ mm).
In figure 14 and table 7, are presented the form and coordinates of the rack-gear tool's profile calculated according to (11)-(14) relations, versus the profile determined by CAD method, previously presented.

Table. 7. Coordinates of points on rack-gear profile

ξ [mm]	η [mm]
-0.00273	-11.07982
0.1355	-10.94476
0.27482	-10.80921
0.41525	-10.67315
0.55682	-10.53655
0.69954	-10.39938
0.84345	-10.2616
0.98856	-10.12318
1.13489	-9.9841
1.28248	-9.84431
⋮	⋮
9.9466	-0.6866
9.95775	-0.61118
9.96761	-0.53545
9.97618	-0.45946
9.98344	-0.38323
9.9894	-0.30681
9.99403	-0.23024
9.99735	-0.15356
9.99934	-0.0768
10	0

6. CONCLUSIONS

The graphical method, developed based on the design environment CATIA capabilities has the advantage to use very versatile software and allow a great originality of the numerical results.
Is eliminated the emerging of the errors due to the passing curves which may be regarded, erroneous, as profile zones.

Fig. 14. Rack-gear for sprocket generation

The *Tachet* mechanism used in the problem solving is universal, excepting the profiles with singular points.

The profiling precision of rack-gear tool is comparable with results obtained by analytical methods, based on which was created dedicated software, in present paper, in Java language.

Using the capabilities of the CATIA design environment, is easy to draw the interference trajectories between the profile to be generated and the generating tool and in the same time the gearing line, only by changing the element regarding which is defined the contact point between the two conjugated profiles.

The proposed validation issue may be extended to the others generating process by enveloping, by the rolling method (generation with gear-shaped tool or generation with worm mill tool).

ACKNOWLEDGEMENTS

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